Physics on the boundary between Quantum Mechanics and Gravity

Philipp Köhler

University of Vienna

Quantum Fields, Gravity and Information
5.4.2013
1 Introduction
   • Dynamical Reduction Models

2 Collapse Models and Spontaneous Localization
   • CSL Model
   • Mass in CSL Model
   • Diosi’s QMUDL
   • Schrödinger Newton Equation

3 Conclusions
Why introduce something new into Quantum mechanics?

- Schrödinger Cat States
- Description of measurement process
- Linearity of quantum mechanics vs Nonlinearity of General Relativity
Measurement in quantum mechanics

Density Matrix

\[ \rho \text{ before measurement} \rightarrow \frac{P_n \rho P_n}{\text{Tr}(P_n \rho P_n)} \text{ after measurement} \]

Measurement Problem

\[ |s_n\rangle \otimes |A_1\rangle \rightarrow |s_n\rangle \otimes |A_n\rangle \]

Problem:

\[ \frac{1}{\sqrt{2}} (|s_n\rangle + |s_l\rangle) \rightarrow \frac{1}{\sqrt{2}} (|s_n\rangle \otimes |A_n\rangle + |s_l\rangle \otimes |A_l\rangle) \]

Entanglement between Apparatus and measured system.
Interpretation:

Measuring either $|s_n\rangle \otimes |A_n\rangle$ or $|s_l\rangle \otimes |A_l\rangle$ with probability $\frac{1}{2}$.

This leads to two different time evolutions:

- Schrödinger evolution $\rightarrow$ linear, deterministic
- Wavepacket reduction $\rightarrow$ nonlinear, stochastic
Possible ways to deal with this problem [Bassi, 2003]:

- Incompleteness
- formal Completeness
- with different Individuals
- with identical Individuals
- Two dynamical principles
- Unifyied dynamics
Dynamical reduction models:
Simple example:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|x_1\rangle + |x_2\rangle)$$

$$|\psi\rangle\langle\psi| = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

Look for "collapsed" system

$$\frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \rightarrow \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Also evolution has to give the right mixture, i.e. individual parts have to have definite macroscopic properties.
Linear, stochastic

\[ i\hbar \partial_t |\psi(t)\rangle = \sum_i P_i V_i(t) |\psi(t)\rangle \]

\[ \langle\langle V_i(t) \rangle\rangle = 0 \]

\[ \langle\langle V_i(t)V_j(t)\rangle\rangle = \gamma \delta_{ij} \delta(t - t') \]

\[ \partial_t \langle x_i | \rho(t) | x_j \rangle = \gamma (\delta_{ij} - 1) \langle x_i | \rho(t) | x_j \rangle \]

BUT

\[ \langle \psi(t) | P_i | \psi(t) \rangle = \langle \psi(0) | P_i | \psi(0) \rangle \]

Nonlinear, deterministic

Ensembles \( E_1, E_2 \)

\[ E_1 \{ a_i, |\phi_i\rangle \} \quad E_2 \{ b_i, |\varphi_i\rangle \} \]

Purification

\[ \sum_i a_i |\phi_i\rangle \otimes |\eta_i\rangle = \sum_i b_i |\varphi_i\rangle \otimes |\zeta_i\rangle \]

Two regions \( X_1 \rightarrow \phi, \varphi, X_2 \rightarrow \eta, \zeta \)

Create ensembles by measuring in \( X_2 \). Due to No-Signalling \( \rightarrow \) only linear operations allowed! [Gisin, 1989]
Collapse Models and Spontaneous Localization

Collapse Models are Nonlinear and Stochastic.

Bassi, Dürr, Hinrichs 2013: [Bassi, 2013]

"Collapse models are the only possible nonlinear extensions of the Schrödinger equation, compatible with the no-faster-than-light assumption."

Only equations of type

\[ d\psi_t = \left( -iHdt + \sum_{k=1}^{n} (L_k - l_{k,t})dW_{k,t} - \frac{1}{2} \sum_{k=1}^{n} (L_k^\dagger L_k + 2l_{k,t}L_k + |l_{k,t}|^2)dt \right)\psi_t \]

\[ l_{k,t} = \frac{1}{2} \langle \psi_t | (L_k^\dagger + L_k) | \psi_t \rangle \]

are allowed.
The CSL Model

\[ \partial_t \rho(t) = -\frac{i}{\hbar} [H, \rho(t)] + \gamma \int d^3 x N(x) \rho(t) N(x) - \frac{\gamma}{2} \int d^3 x \{ N^2(x), \rho(t) \} \]

where \( N(x) \) denotes the preferred basis

\[ N(x) = \int d^3 y g(y - x) a^\dagger(y) a(y) \quad g(x) = \left( \frac{\alpha}{2\pi} \right)^{\frac{3}{2}} e^{-\frac{\alpha}{2} x^2} \]

Simple model:

\[ \partial_t \rho(t) = \gamma \left( \frac{\alpha}{4\pi} \right)^{\frac{3}{2}} \sum_k \left[ \sum_i N_i^{(k)} \rho(t) N_i^{(k)} - \frac{1}{2} \sum_i \{ N_i^{(k)}^2, \rho(t) \} \right] \]

\[ \langle n_1^{(k)}, \ldots | \rho(t) | m_1^{(k)}, \ldots \rangle = e^{-\gamma^2 \left( \frac{\alpha}{4\pi} \right)^{\frac{3}{2}} \sum_{i,k} (n_i^{(k)} - m_i^{(k)})^2} \langle n_1^{(k)}, \ldots | \rho(0) | m_1^{(k)}, \ldots \rangle \]
Question:
Meaning of Parameters?

Replace Number operator with Mass Density operator

\[ M(x) = \sum_k m_k N_k(x) \]

\[ \partial_t \rho(t) = -\frac{i}{\hbar} [H, \rho(t)] + \frac{\gamma}{m_0^2} \int d^3x M(x) \rho(t) M(x) - \frac{\gamma}{2m_0^2} \int d^3x \{M^2(x), \rho(t)\} \]

Thus relate collapse to the mass density of the particles. Still two arbitrary parameters needed.
One step further:

Diosi’s proposition of collapse due to gravity.
(QMUDL: quantum mechanics with universal density localization)

Introduce mass density

\[ f(x) = \frac{M}{V} \theta(R - |x' - x|) \]

\[ \partial_t \rho(t) = -\frac{i}{\hbar} [H, \rho(t)] - \frac{G}{2\hbar} \int \int \frac{d^3x_1 d^3x_2}{|x_1 - x_2|} [f(x_1), [f(x_2), \rho(t)]] \]

for description of decoherence processes due to gravity.
Assume Gravitational Potential as cause for Collapse

\[ U(x) = -G \int \frac{f(x_1)f(x_2)}{|x_1 - x_2|} \, dx_1 \, dx_2 \]

This leads to

\[ \langle x' | \rho(t) | x'' \rangle = e^{\Gamma(|x' - x''|)t} \langle x' | \rho(0) | x'' \rangle \]

where

\[ \Gamma(|x' - x''|) = \frac{1}{\hbar}(U(0) - U(|x' - x''|)) \]
Assume Gravitational Potential as cause for Localization

Modify Schrödinger Equation:

\[ i\hbar \partial_t \psi = \left( -\frac{\hbar^2}{2m} \Delta - Gm^2 \int \frac{|\psi|^2}{|x - y|} d^3y \right) \psi \]

\[ \Delta \Phi = 4\pi Gm|\psi|^2 \]

Nonlinear Integro-Differential Equation!
Scaling of Schrödinger Newton

\[ m_{\mu} = \mu m \]
\[ t_{\mu} = \mu^5 t \]
\[ r_{\mu} = \mu^3 r \]
\[ \psi_{\mu} = \mu^9 \psi \]
Numerical solutions: Solitons!

Abbildung: Solitonic solution of the Schrödinger Newton equation. Step size: $r = 10^{-8} m$, Blue: Gauss Packet at $t = 0$, Red: Schrödinger Newton solution after 20000s, Yellow: Schrödinger solution after 20000s.
Abbildung: Solution of the Schrödinger Newton equation. Step size: \( r = 10^{-14} \text{m} \), Blue: Gauss Packet at \( t = 0 \), Red: Schrödinger Newton solution after \( 10^{-10} \text{s} \), Yellow: Schrödinger solution after \( 10^{-10} \text{s} \).
Conclusions

- Framework beyond quantum mechanics.
- Is the only feasible Ansatz that does not contradict the No-faster-than-light assumption.
- Allows explanation of the ad-hoc collapse of the wavefunction without introducing a second time evolution.
- Introduced parameters may be linked to Gravity.
- Experimental tests may be possible to distinguish these models from standard quantum mechanics.
Thank you for your attention!

References: