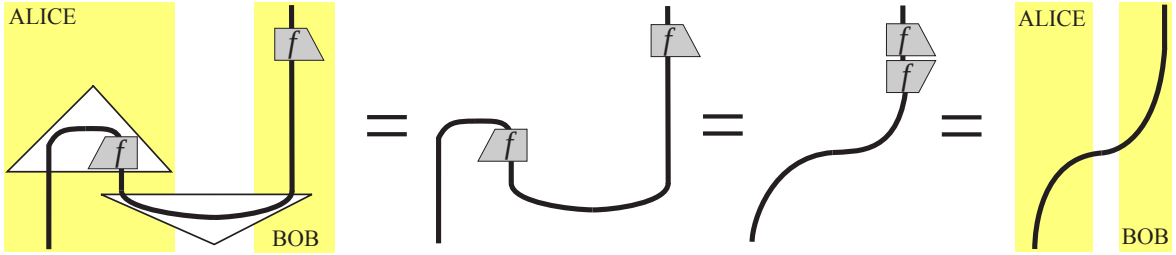
quantum information flow

Bob Coecke – Quantum Group - Computer Science - Oxford University



Samson Abramsky & BC (2004) *A categorical semantics for quantum protocols*. Logic in Computer Science '04. arXiv:quant-ph/0402130 BC & **Ross Duncan** (2008, 2011) *Interacting quantum observables: categorical algebra and diagrammatics*. New Journal of Physics **13**, 043016. arXiv:0906. 4725

FORTHCOMING: textbook with **Aleks Kissinger** on a purely diagrammatic presentation of basic quantum information

Overview / general idea:

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- Quantum informatic systems and operations and computations thereon admit diagrammatic calculus.

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  - Bipartite entanglement generated (Selinger’08)
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- The mathematical backbone is category theory.

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. . . isn't it just tedious abstract nonsense?

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Symmetric Monoidal Categories are everywhere!

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$$A \xrightarrow{f} B \qquad A \xrightarrow{f'} B \qquad A \xrightarrow{f''} B$$

be boiling, frying, baking. *States* are *processes*

$$\mathrm{I} := \text{unspecified} \xrightarrow{\psi} A.$$

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$$X \xrightarrow{\mathbf{1}_X} X$$

be doing nothing. We have $\mathbf{1}_Y \circ \xi = \xi \circ \mathbf{1}_X = \xi$.

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$$A \otimes D \xrightarrow{f \otimes h} B \otimes E$$

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$$A \otimes D \xrightarrow{f \otimes h} B \otimes E$$

be boiling potato while frying carrot. Let

$$C \otimes F \xrightarrow{x} M$$

be mashing spice-cook-potato and spice-cook-carrot.

5. Total *process*:

$$A \otimes D \xrightarrow{f \otimes h} B \otimes E \xrightarrow{g \otimes k} C \otimes F \xrightarrow{x} M = A \otimes D \xrightarrow{x \circ (g \otimes k) \circ (f \otimes h)} M.$$

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7. *Laws governing recipes*:

$$(\mathbf{1}_B \otimes g) \circ (f \otimes \mathbf{1}_C) = (f \otimes \mathbf{1}_D) \circ (\mathbf{1}_A \otimes g)$$

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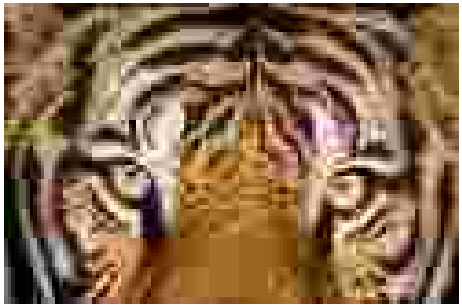
$\Rightarrow$ **Symmetric Monoidal Category**

— *Why does a tiger have stripes and a lion doesn't?* —



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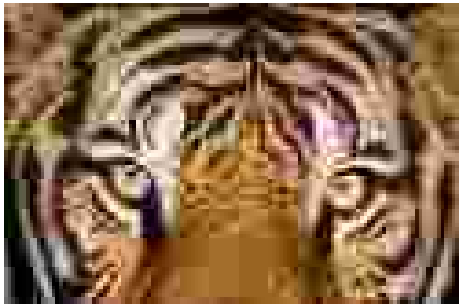


prey $\otimes$ predator $\otimes$ environment

$\downarrow$ hunt

dead prey $\otimes$ eating predator

BOXES AND WIRES

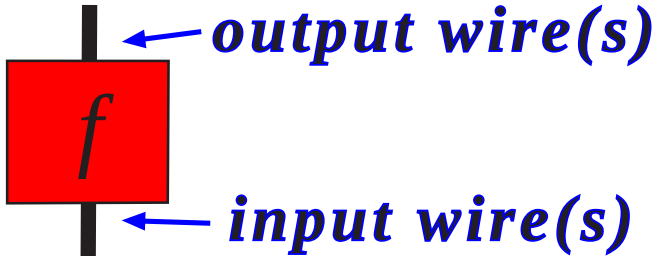
Roger Penrose (1971) *Applications of negative dimensional tensors*. In: Combinatorial Mathematics and its Applications, 221–244. Academic Press.

André Joyal and Ross Street (1991) *The Geometry of tensor calculus* I. Advances in Mathematics **88**, 55–112.

Bob Coecke and Eric Oliver Paquette (2011) *Categories for the practicing physicist*. In: New Structures for Physics, B. Coecke (ed), Springer-Verlag. `arXiv:0905.3010`

— *wire and box language* —

Box :=



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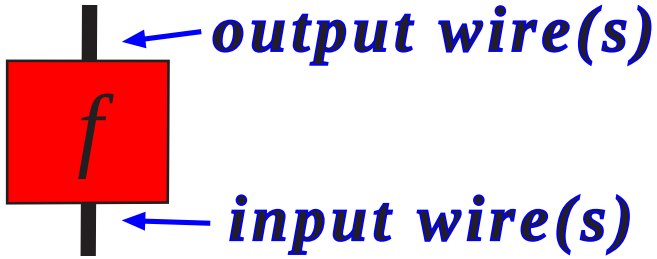
Box :=



Interpretation: wire := system ; box := **process**

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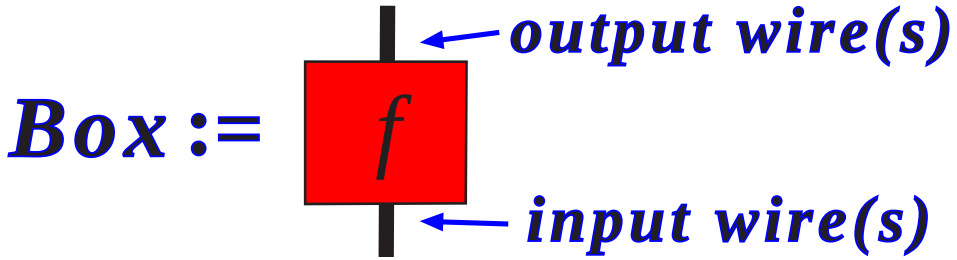

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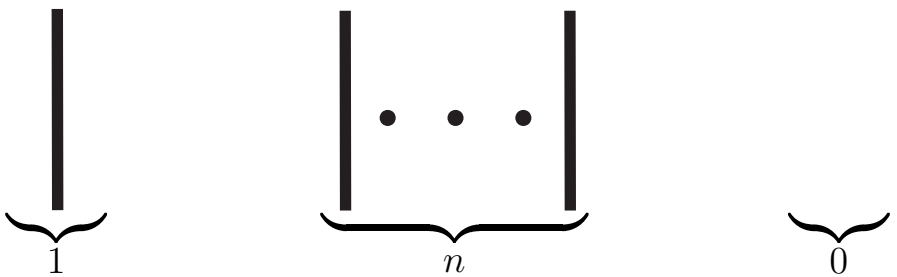
one system: n *sub*systems: no system:



— *wire and box games* —

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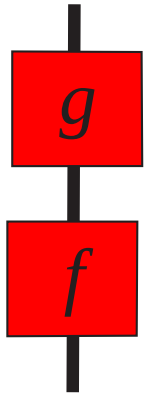
sequential or *causal* or *connected* composition:

$g \circ f \equiv$



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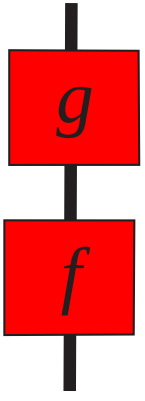
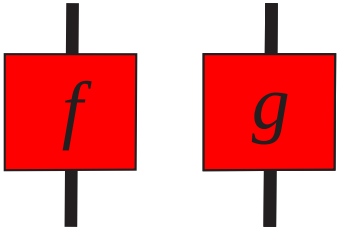


parallel or *acausal* or *disconnected* composition:

$f \otimes g \equiv$



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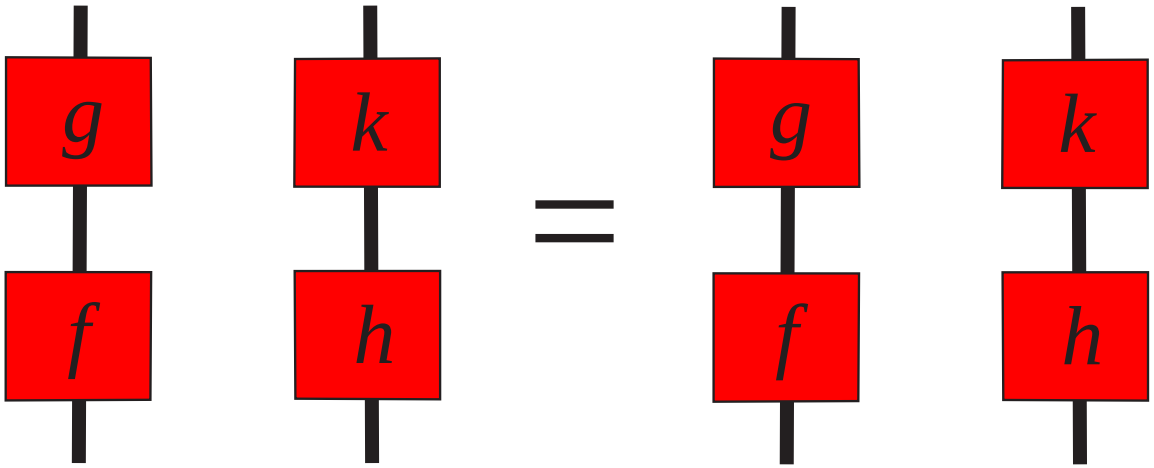

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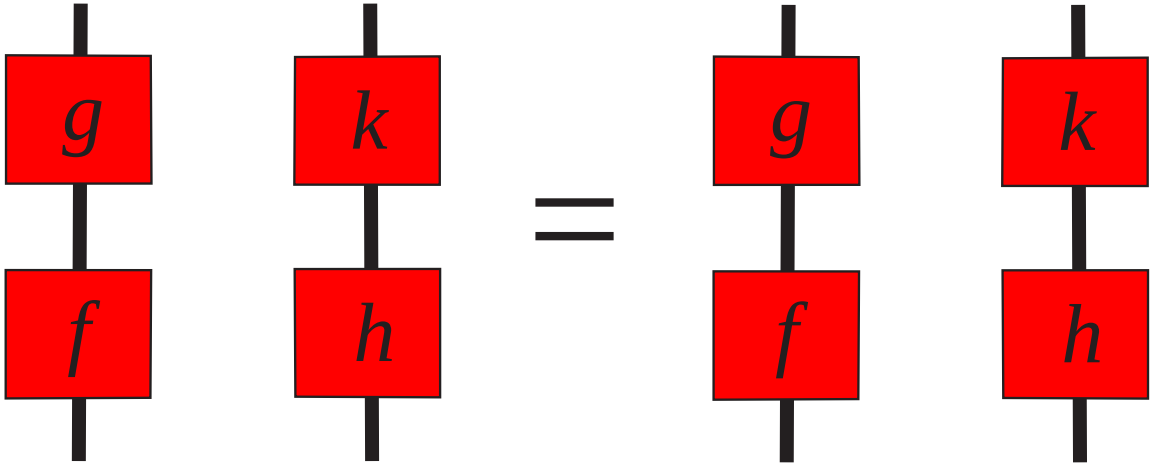
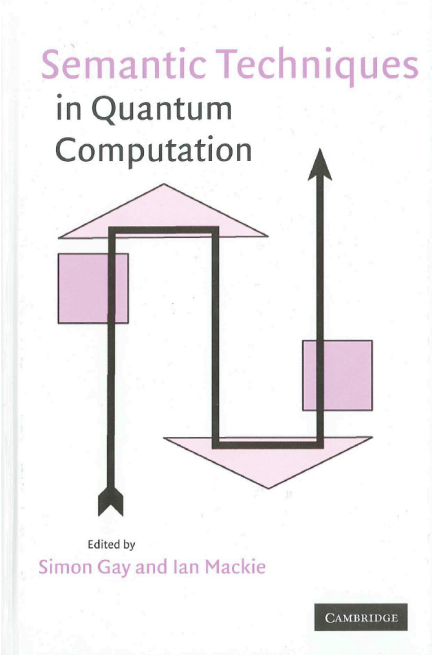


peel potato and then fry it,
while,
clean carrot and then boil it

=

peel potato while clean carrot,
and then,
fry potato while boil carrot

MINIMAL QUANTUM PROCESS LANGUAGE



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WHY?

von Neumann: only used *it* since *it* was ‘available’.

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Quantum Computer Scientists: Schrödinger is right!

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Task 1. Investigate which assumptions (i.e. which structure) on $\otimes$ is needed to deduce **physical phenomena**.

Task 2. Investigate wether such an "interaction structure" appear elsewhere in **"our classical reality"**.

Outcome 1a:

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Outcome 1b: Exposing this structure has already helped to **solve open problems elsewhere**. (e.g. $2\times$ ICALP'10)

E.g.: Ross Duncan & Simon Perdrix (2010) *Rewriting measurement-based quantum computations with generalised flow*. ICALP'10.

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Outcome 1b: Exposing this structure has already helped to **solve open problems elsewhere**. (e.g. $2\times$ ICALP'10)

Outcome 1c: **Framework is a simple intuitive (but rigorous) diagrammatic language**, meanwhile adopted by others e.g. Lucien Hardy in arXiv:1005.5164:

"... we join the ***quantum picturalism*** revolution [1]"

---

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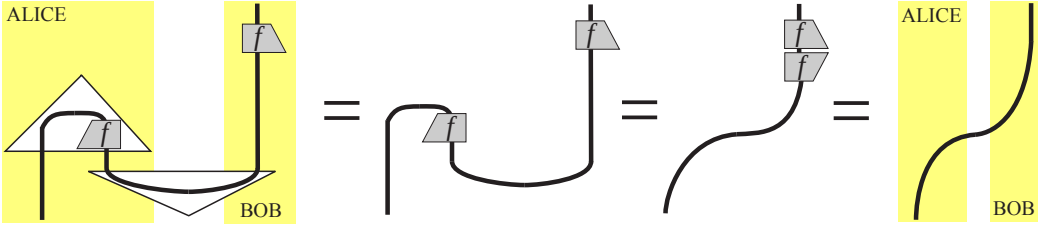
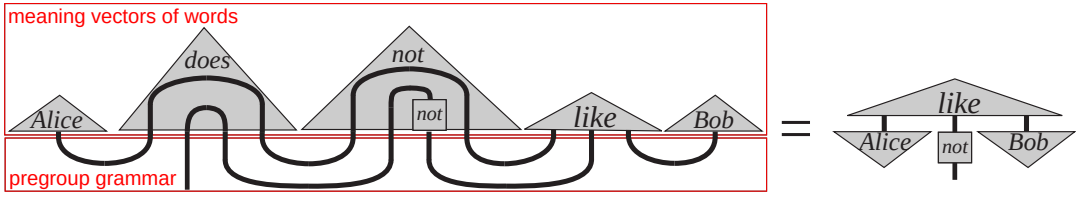
Outcome 2a:

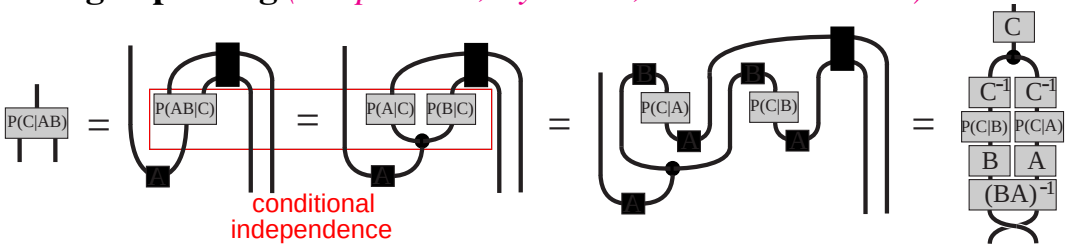
Behaviors of matter *(Abramsky-C; LiCS'04, quant-ph/0402130)* :



Meaning in language *(Clark-C-Sadrzadeh; Linguistic Analysis, arXiv:1003.4394)* :



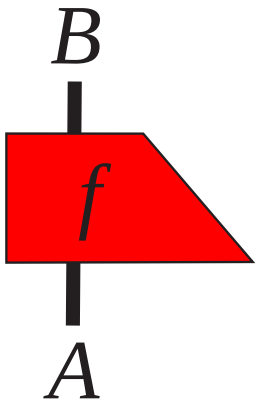
Knowledge updating *(C-Spekkens; Synthese, arXiv:1102.2368)* :



BOXES AND WIRES II

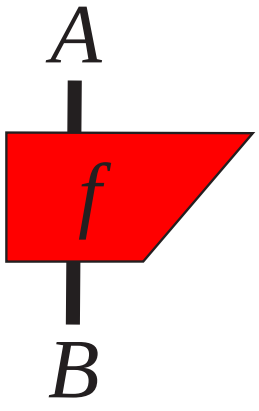
— *quantitative metric* —

$$f : A \to B$$



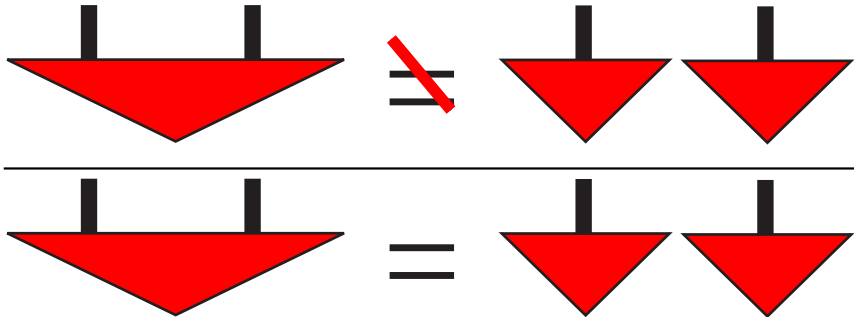
— *quantitative metric* —

$$f^{\dagger} : B \to A$$



— *asserting (pure) entanglement* —

$$\frac{quantum}{classical} =$$



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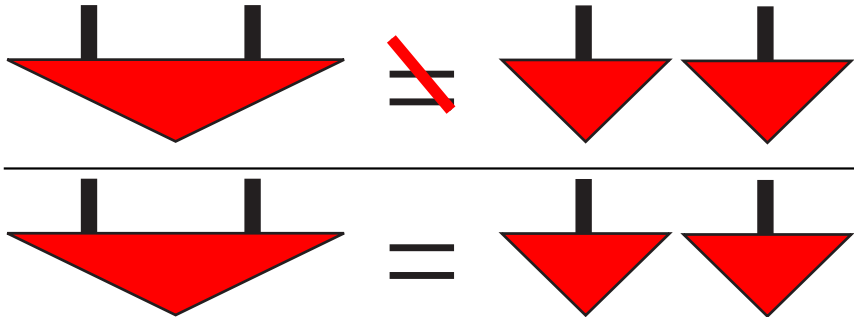


$\Rightarrow$ **introduce 'parallel wire' between systems:**

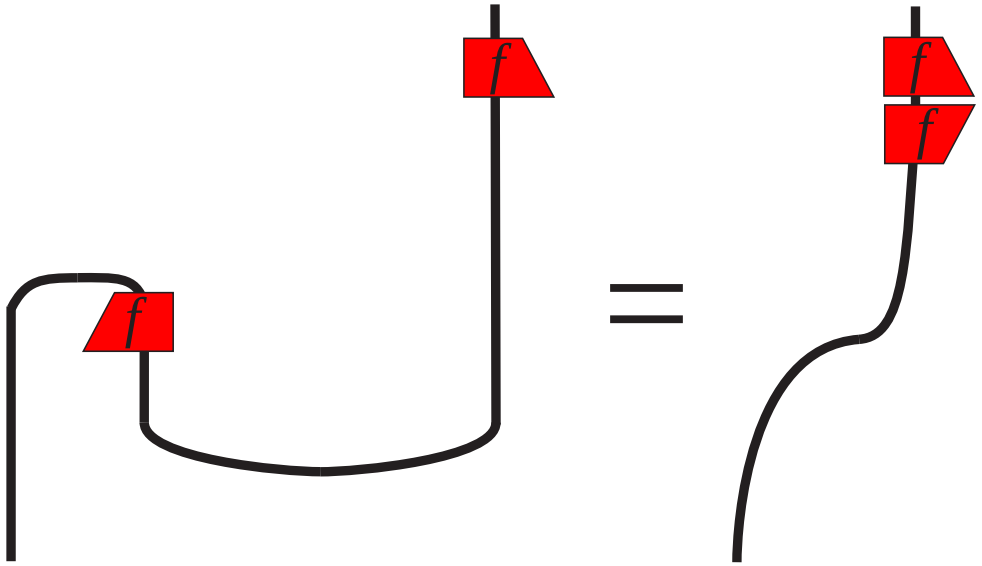
subject to: only topology matters!

— *quantum-like* —

E.g.

Transpose:

Conjugate: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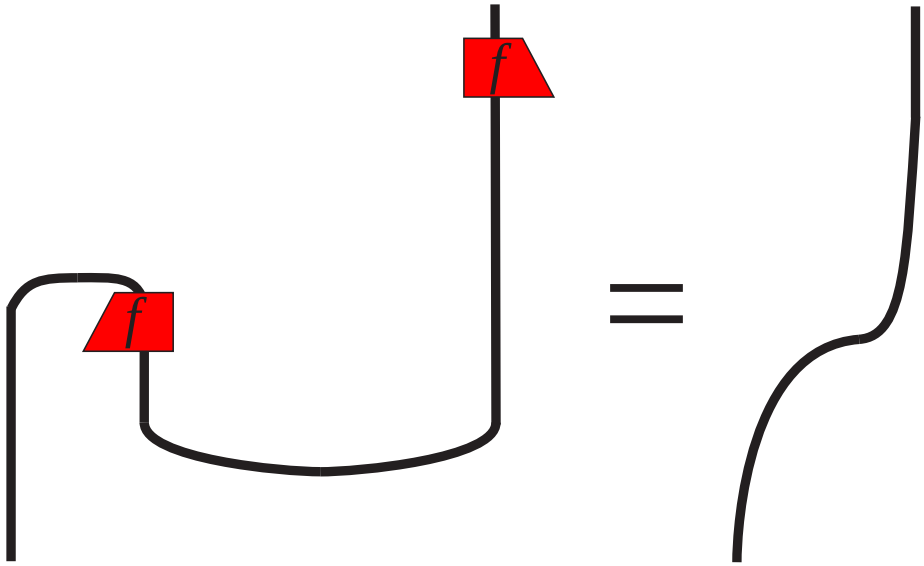
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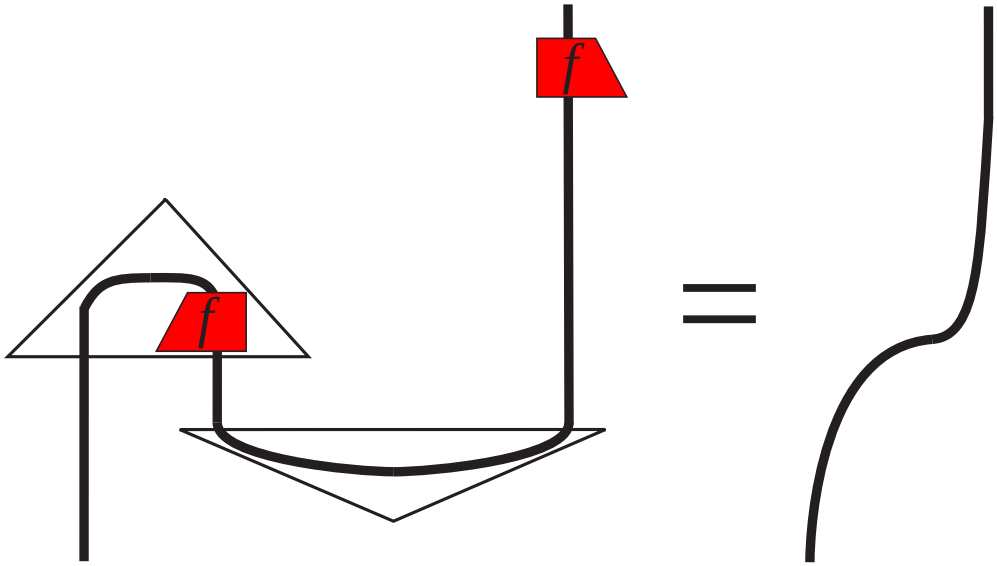
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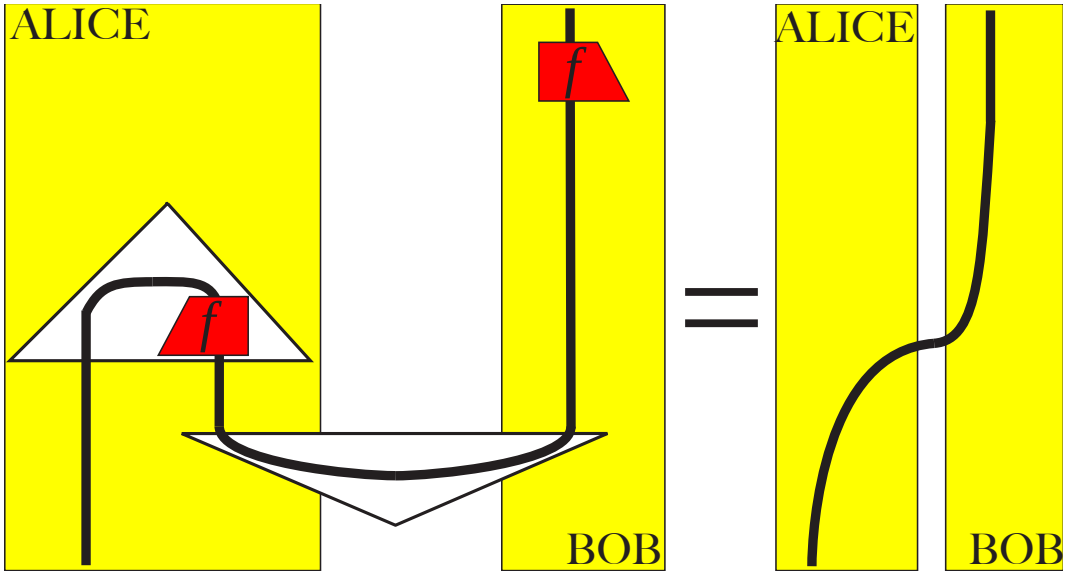
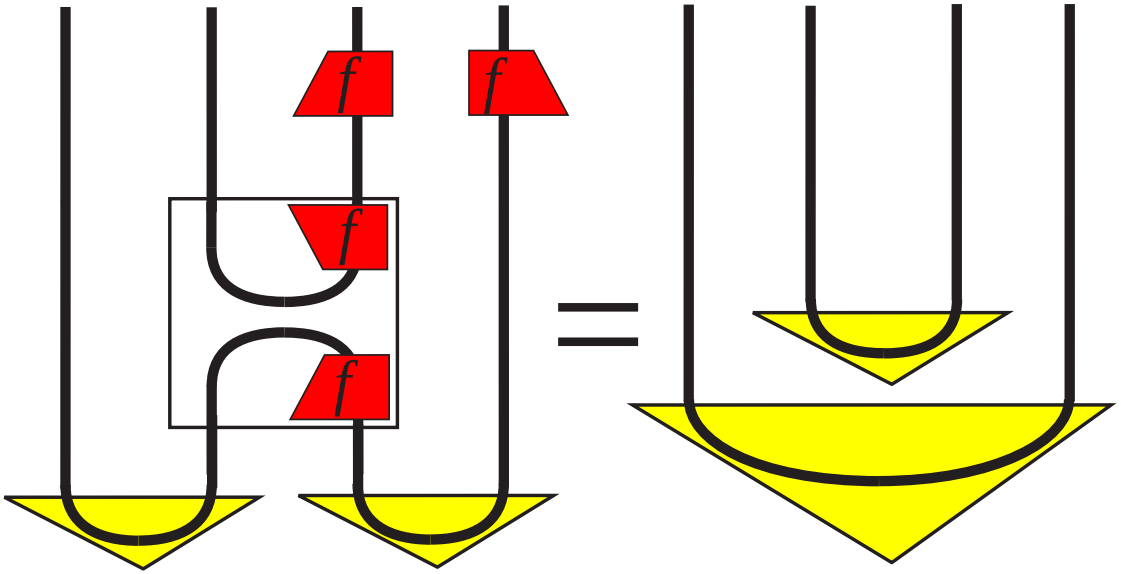
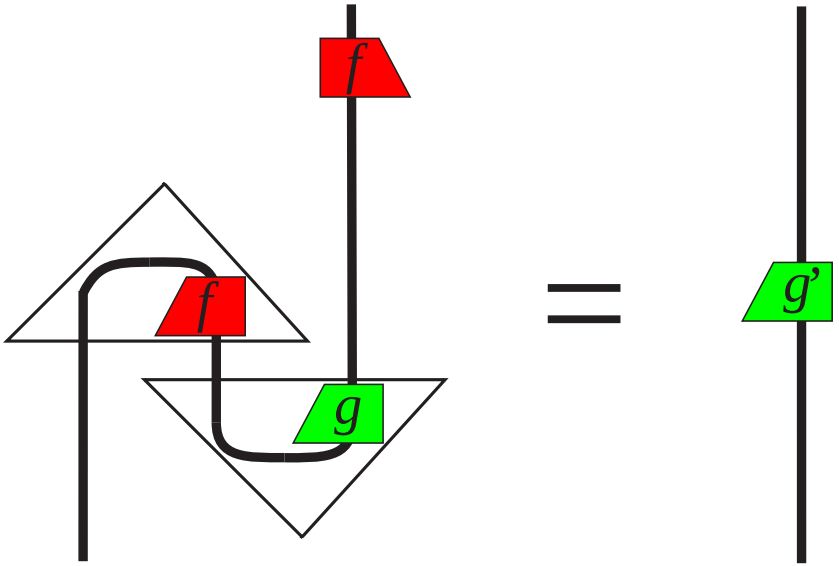


⇒ quantum teleportation



$\Rightarrow$ **Entanglement swapping**



⇒ gate teleportation MBQC

— *symbolically: dagger compact categories* —

Thm. [Kelly-Laplaza '80; Selinger '05] *An equational statement between expressions in dagger compact categorical language holds if and only if it is derivable in the graphical notation via homotopy*.

Thm. [Hasegawa-Hofmann-Plotkin; Selinger '08] *An equational statement between expressions in dagger compact categorical language holds if and only if it is derivable in the dagger compact category of finite dimensional Hilbert spaces, linear maps, tensor product and adjoints*.

— *symbolically: dagger compact categories* —

In words: *Any equation involving:*

- *states, operations, effects*
- *unitarity, adjoints (e.g. self-adjoint), projections*
- *Bell-states/effects, transpose, conjugation*
- *inner-product, trace, Hilbert-Schmidt norm*
- *positivity, completely positive maps, ...*

holds in quantum theory if and only if it can be derived in the graphical language via homotopy.

What are these diagrams in ordinary QM?

- Step 1: only maps (no vectors, numbers)
- Step 2: cast Dirac in 2D
- Step 3: summations $\mapsto$ topological entity
  - E.g. $\sum_i |ii\rangle \mapsto$ 

. . . glass board tutorial

A SLIGHTLY DIFFERENT LANGUAGE FOR NATURAL LANGUAGE MEANING

BC, Mehrnoosh Sadrzadeh & Stephen Clark (2010) *Mathematical foundations for a compositional distributional model of meaning*. `arXiv:1003.4394`

BC (2012) *The logic of quantum mechanics – Take II*. `arXiv:1204.3458`

— *the from-words-to-a-sentence process* —

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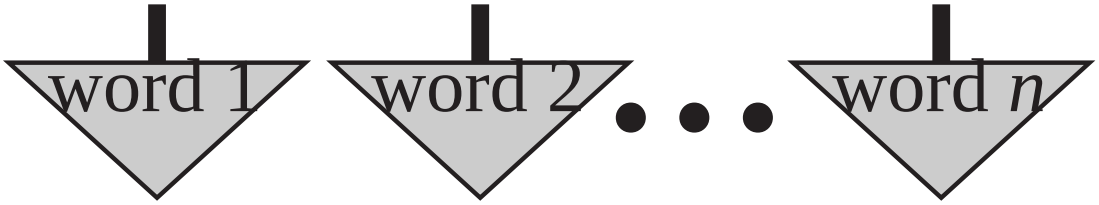
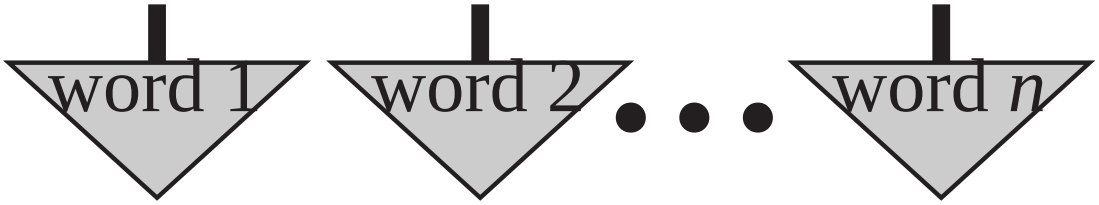
Consider meanings of **words**, e.g. as vectors (cf. Google):



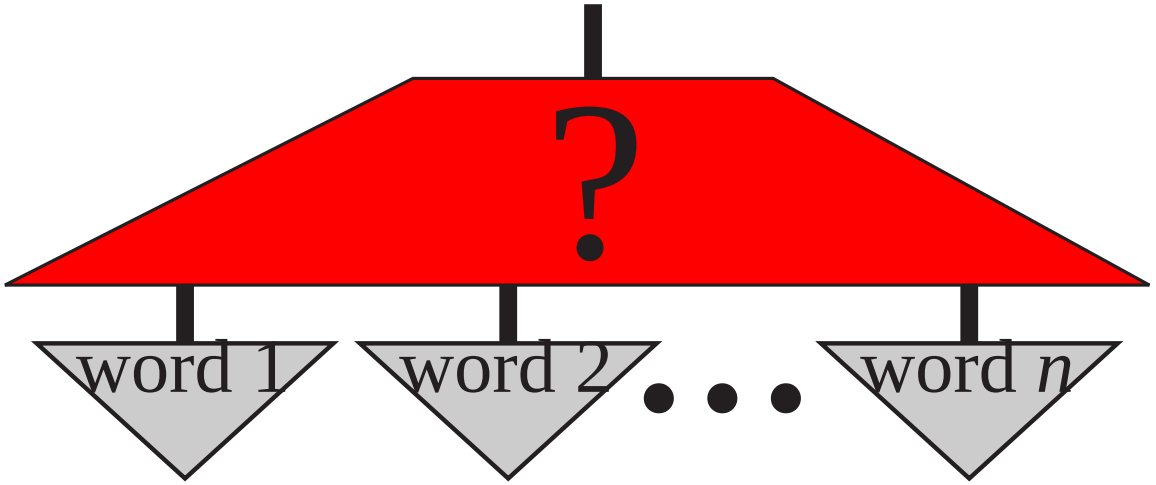
— *the from-words-to-a-sentence process* —

What is the meaning the **sentence** made up of these?



— *the from-words-to-a-sentence process* —

I.e. how do we/machines produce meanings of **sentences**?



— *the from-words-to-a-sentence process* —

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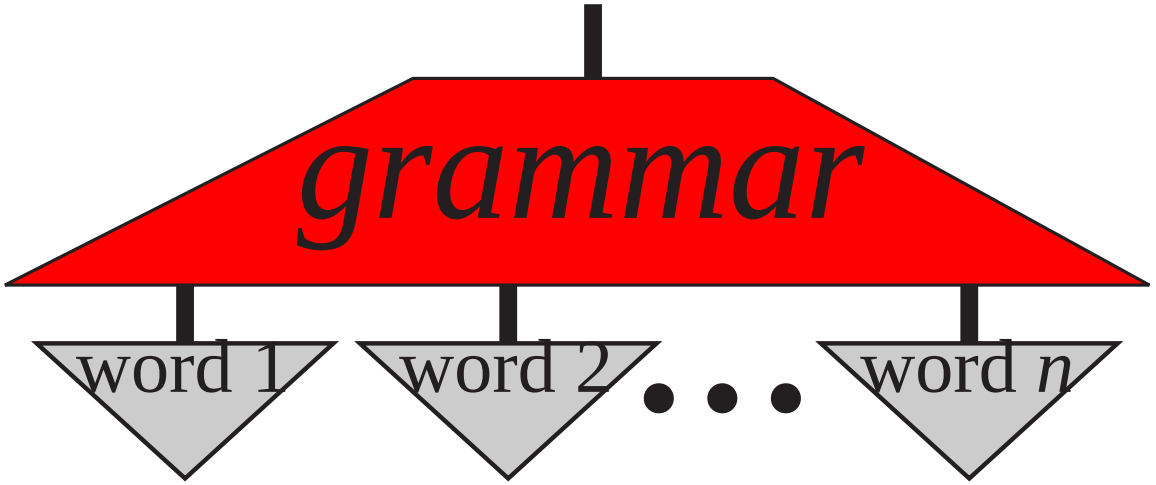
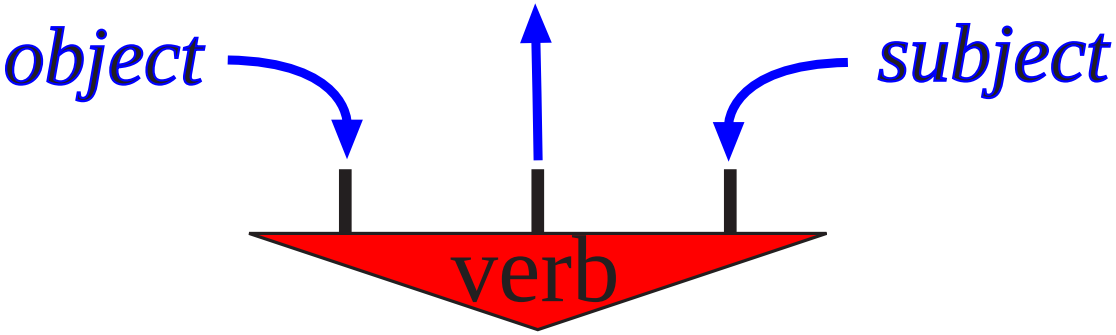


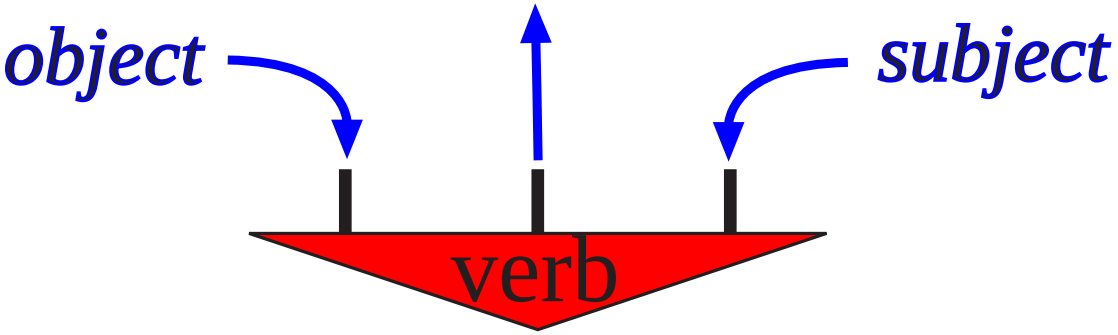
Gerald Gazdar (1996) *Paradigm merger in natural language processing*. In: Computing tomorrow: future research directions in computer science, eds., I. Wand and R. Milner, Cambridge University Press.

Information flow within a verb:

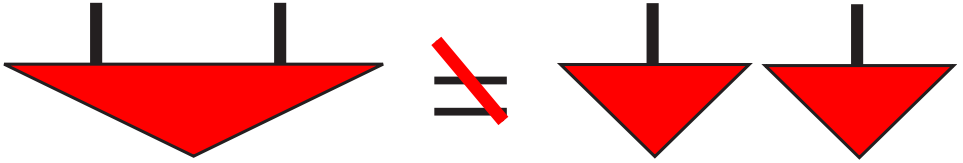


— *the from-words-to-a-sentence process* —

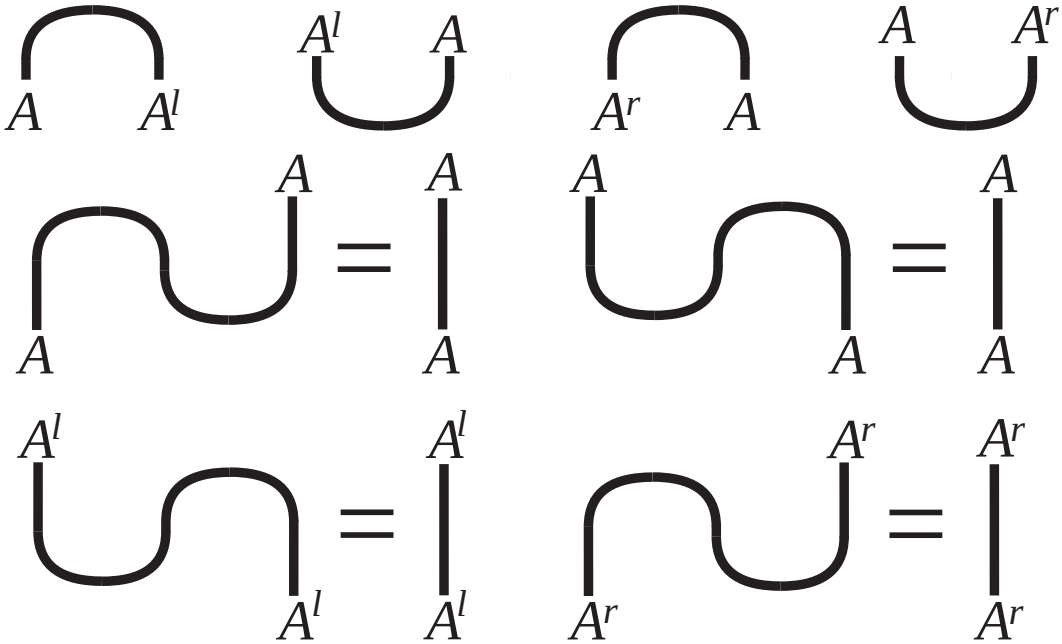
Information flow within a verb:



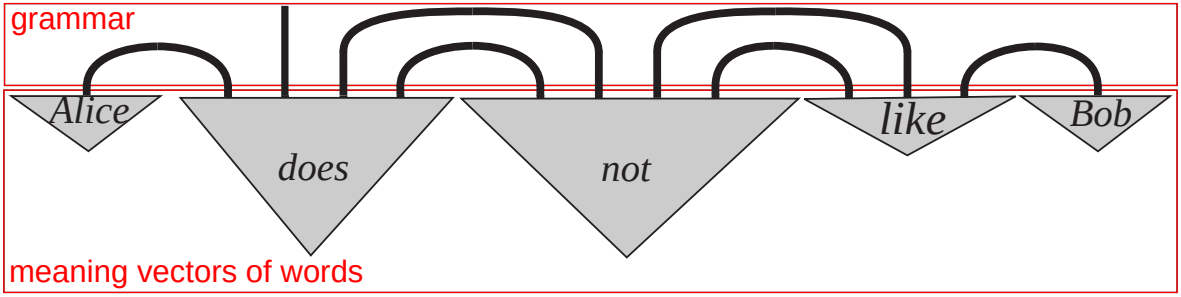
Again we have:



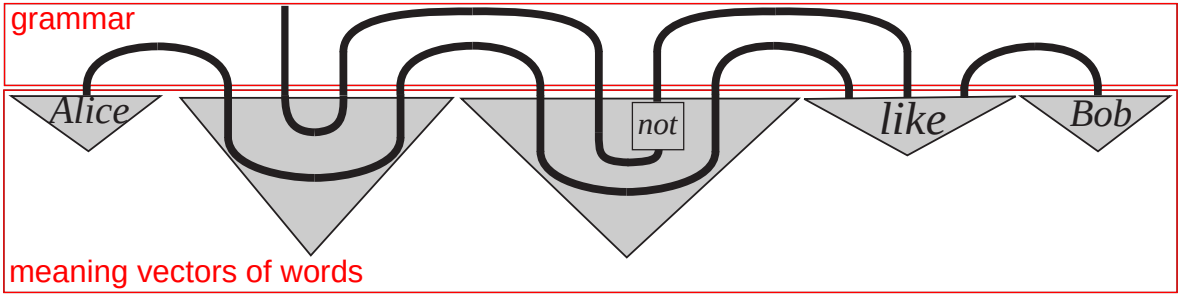
— *grammar as pregroups – Lambek '99* —



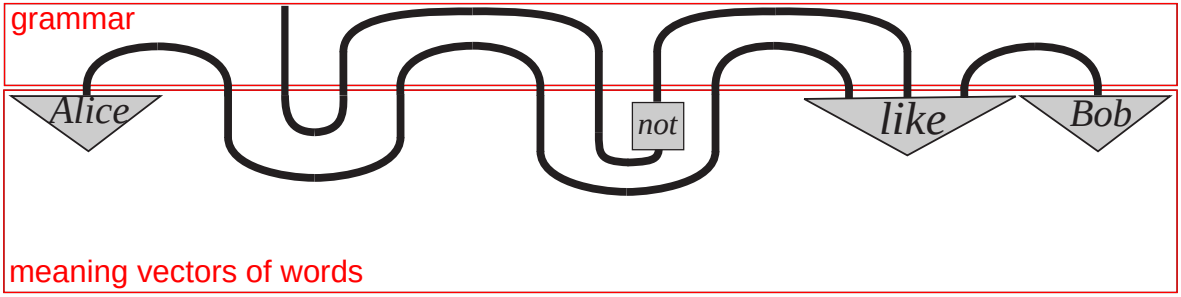
$$— \overrightarrow{Alice} \otimes \overrightarrow{does} \otimes \overrightarrow{not} \otimes \overrightarrow{like} \otimes \overrightarrow{Bob} —$$



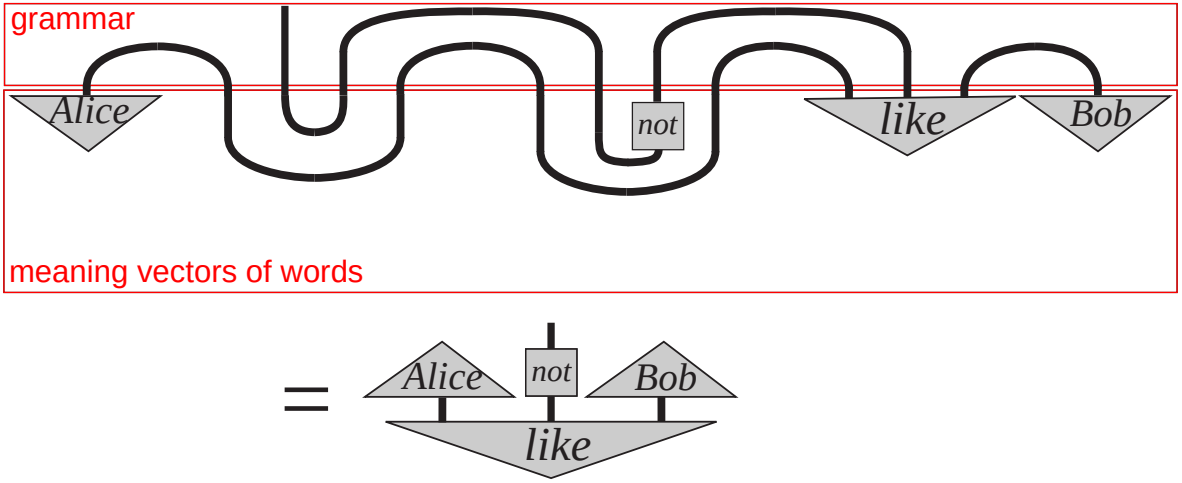
$$— \overrightarrow{Alice} \otimes \overrightarrow{does} \otimes \overrightarrow{not} \otimes \overrightarrow{like} \otimes \overrightarrow{Bob} —$$



$$- \overrightarrow{Alice} \otimes \overrightarrow{does} \otimes \overrightarrow{not} \otimes \overrightarrow{like} \otimes \overrightarrow{Bob} -$$

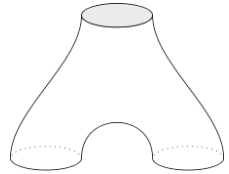


$$- \overrightarrow{Alice} \otimes \overrightarrow{does} \otimes \overrightarrow{not} \otimes \overrightarrow{like} \otimes \overrightarrow{Bob} -$$



— *analogy: quantizing grammar!* —

Topological quantum field theory:

$F : n\mathbf{Cob} \to \mathbf{FVect}_{\mathbb{C}} ::$  $\mapsto V$

Grammatical quantum field theory:

$F : Pregroup \to \mathbf{FVect}_{\mathbb{R}_+} ::$  $\mapsto V$

Louis Crane was the first one to notice this analogy.

UNIVERSAL QUANTUM REASONING

WIRES?

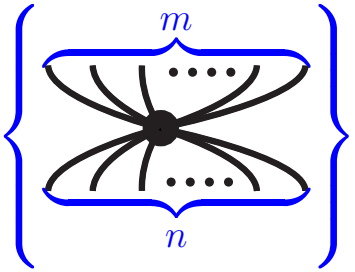
not expressive enough

not expressive enough

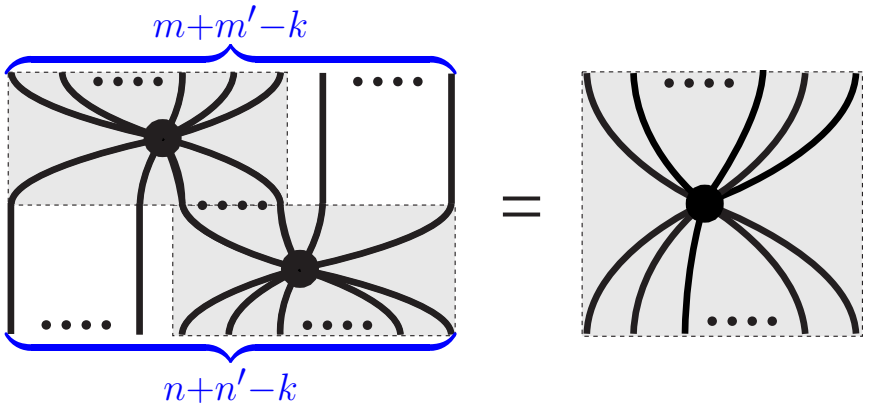
what is expressive enough?

SPIDERS!

— *spiders* —

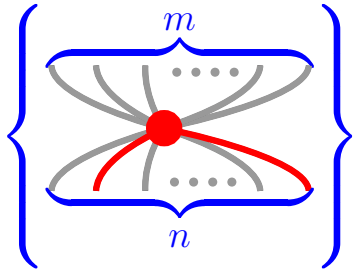
'spiders' $=$ 

such that, for $k > 0$:

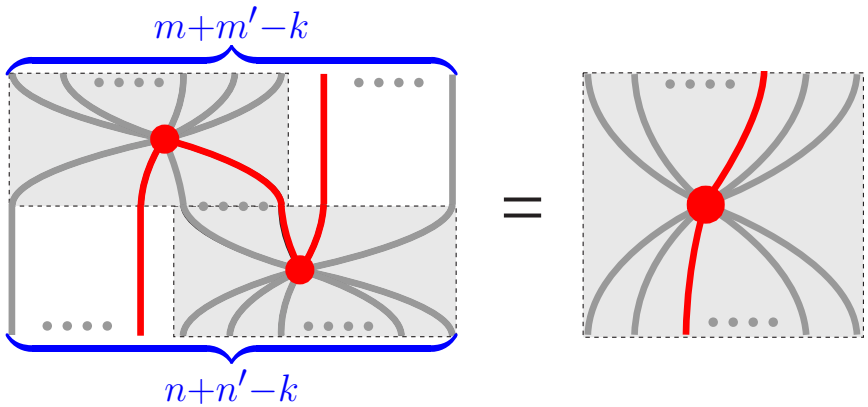


— *spiders* —

'cups/caps' $=$

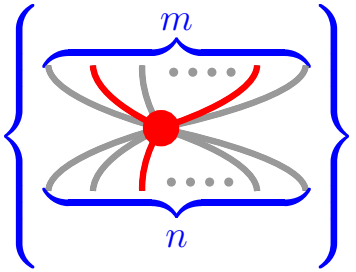


such that, for $k > 0$:

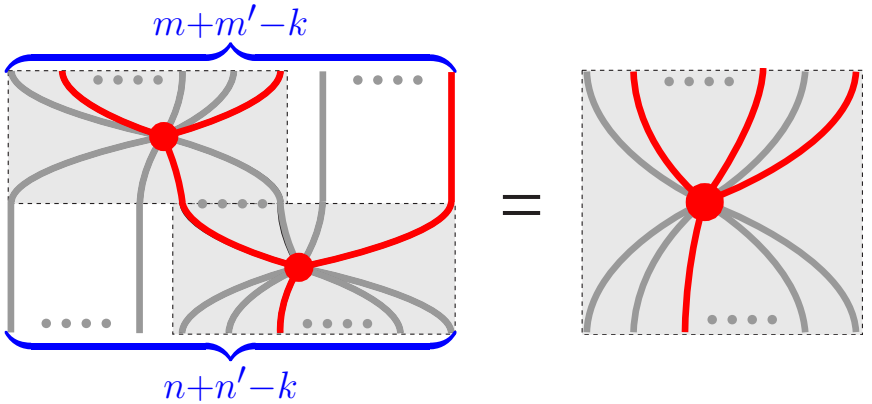


— *spiders* —

'(co-)mult.' $=$

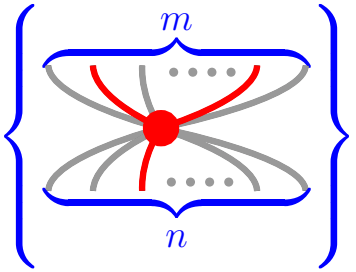


such that, for $k > 0$:

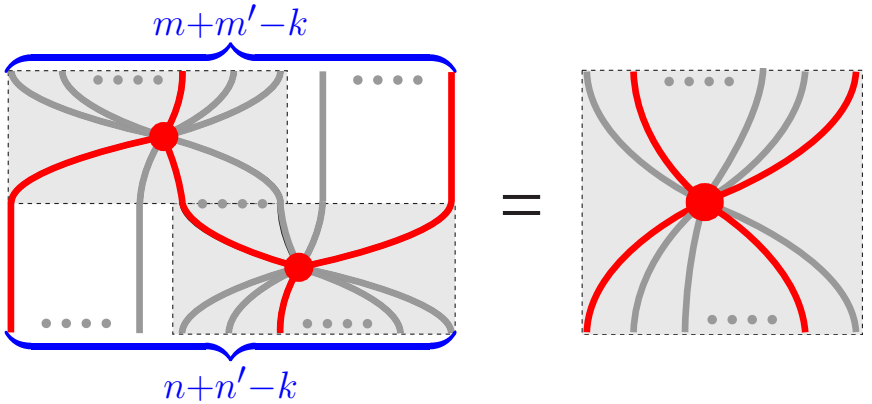


— *spiders* —

'(co-)mult.' $=$



such that, for $k > 0$:



Theorem. The following are the same:

Theorem. The following are the same:

- Linear maps $\{\mathcal{H}^{\otimes n} \to \mathcal{H}^{\otimes m}\}_{n,m}$ as spiders.

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- Orthonormal bases for $\mathcal{H}$.

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- Linear maps $\{\mathcal{H}^{\otimes n} \to \mathcal{H}^{\otimes m}\}_{n,m}$ as spiders.

- Orthonormal bases for $\mathcal{H}$.

- Copying/erasing pairs

$$\Delta : \mathcal{H} \to \mathcal{H} \otimes \mathcal{H} \qquad \epsilon : \mathcal{H} \to \mathbb{C}$$

  that are commutative, dagger Frobenius & special.

To proof:

1a) How does an ONB define a copying/deleting pair?

1b) ... and conversely? (CPV'08 thm)

2a) How does a copying/deleting pair define spiders?

2b) ... and conversely? (TQFT thm)

. . . glass board tutorial

... decorated spiders

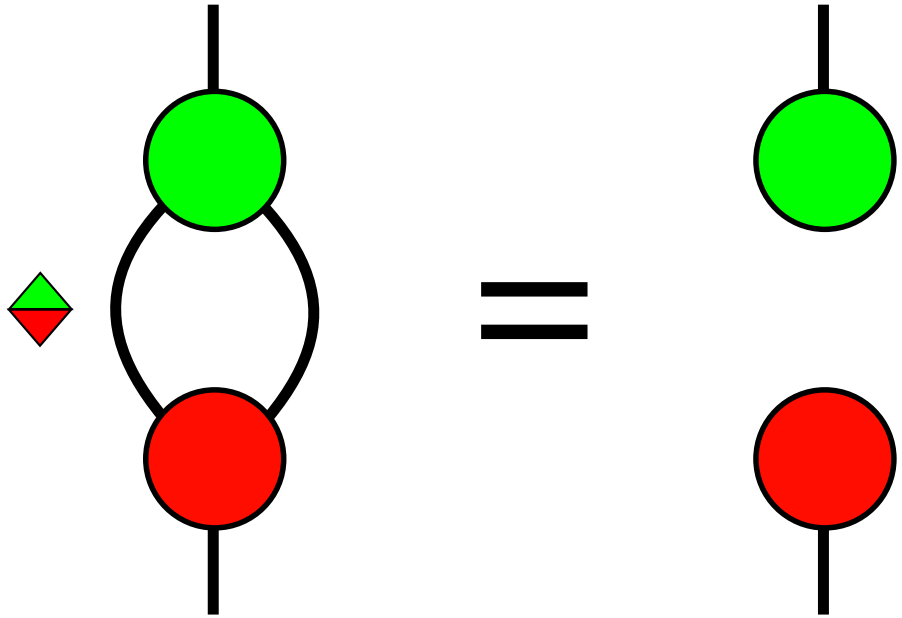
... allergic spiders

UNBIASEDNESS:

“allergic spiders”

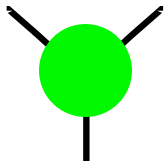
Thm. Unbiasedness $\Leftrightarrow$

Thm. Unbiasedness $\Leftrightarrow$



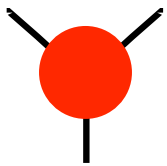
BC & Ross Duncan (2008) *Interacting quantum observables*. ICALP'08 & New Journal of Physics **13**, 043016. `arXiv:0906.4725`

Z-spin:



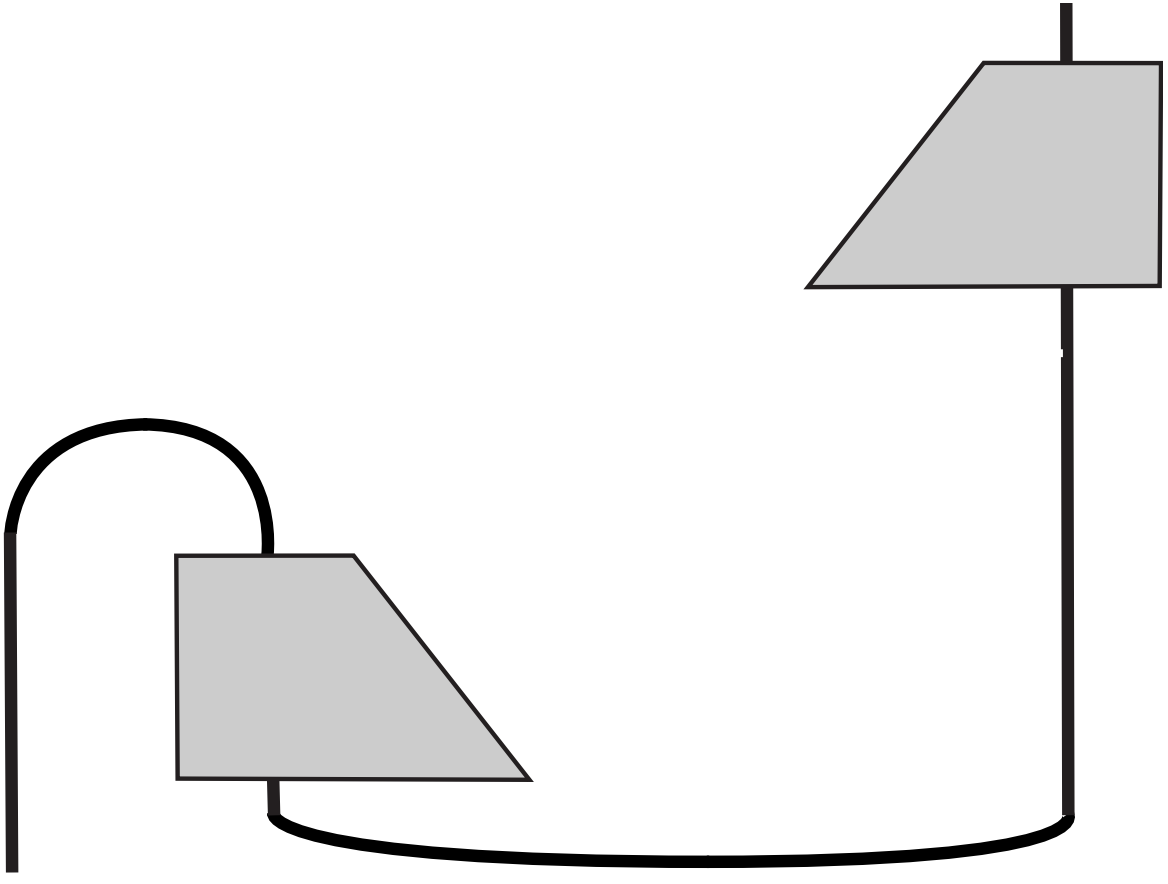
$$\delta_Z : |i\rangle \mapsto |ii\rangle$$

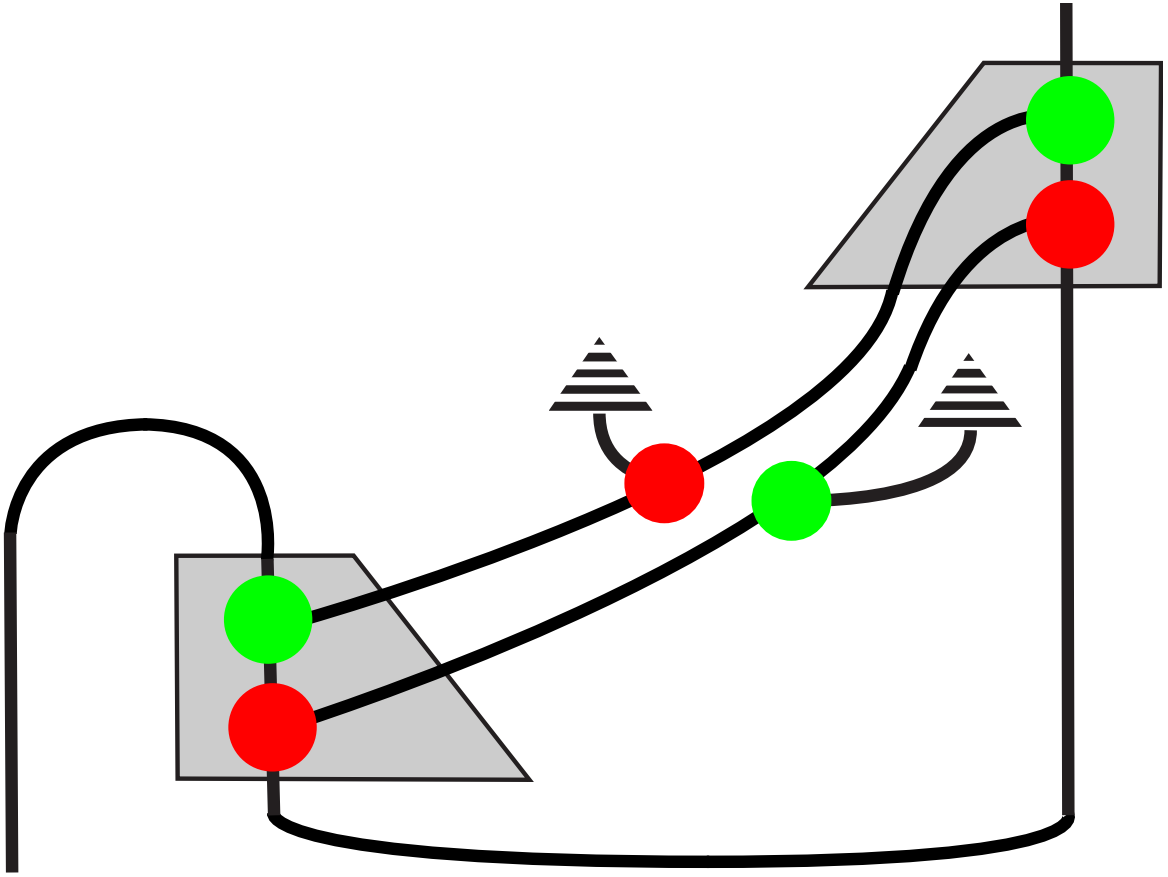
X-spin:

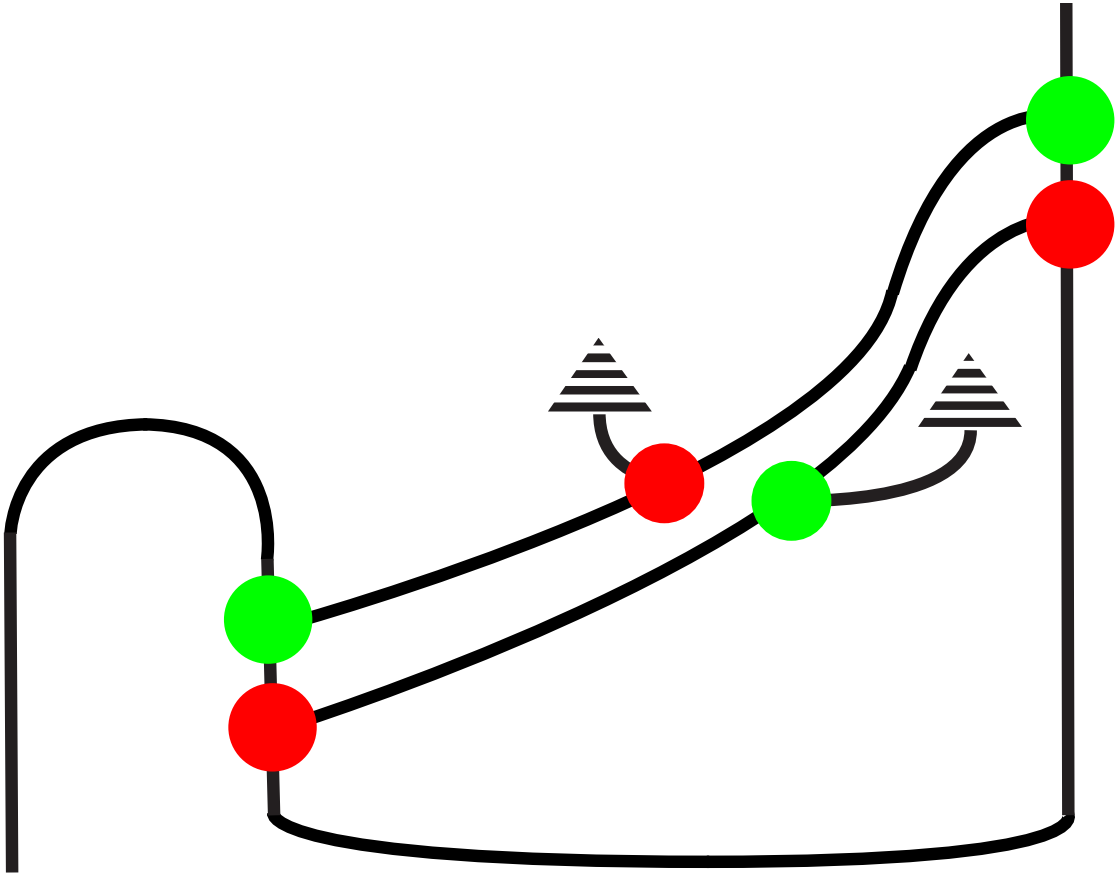


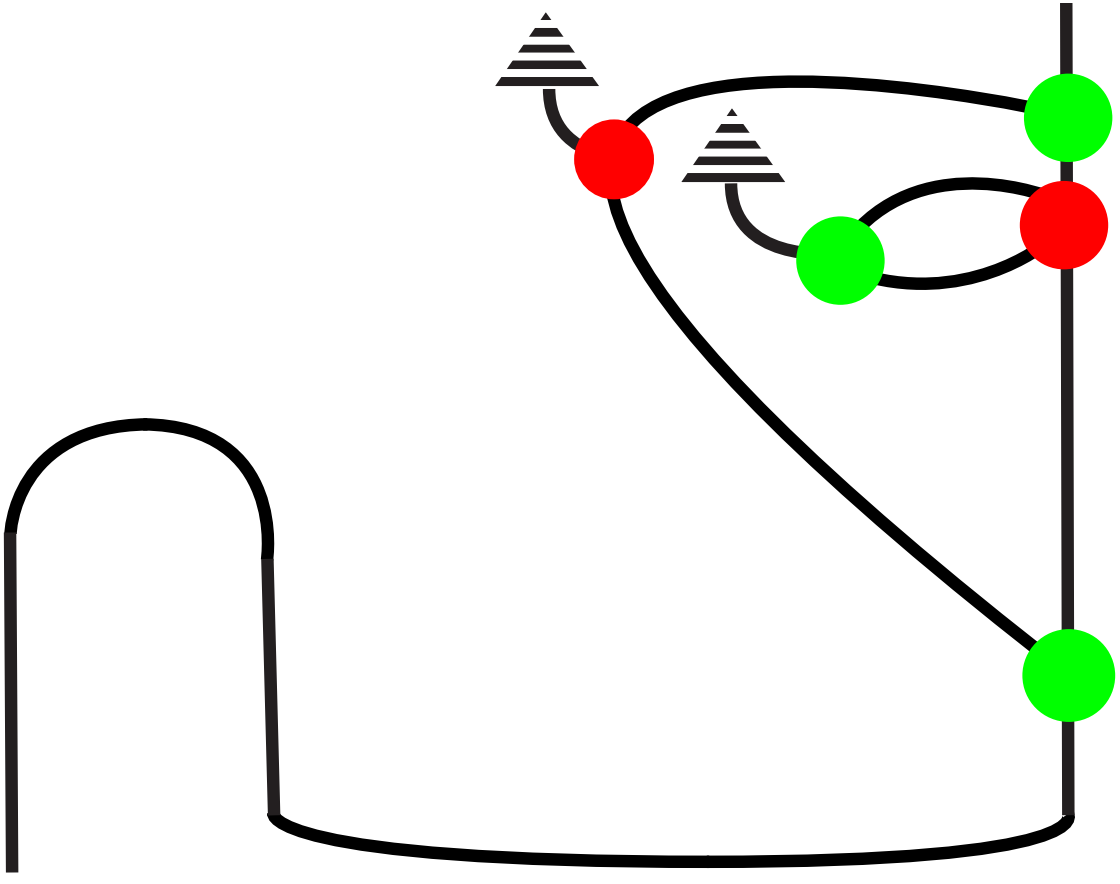
$$\delta_X : |\pm\rangle \mapsto |\pm\pm\rangle$$

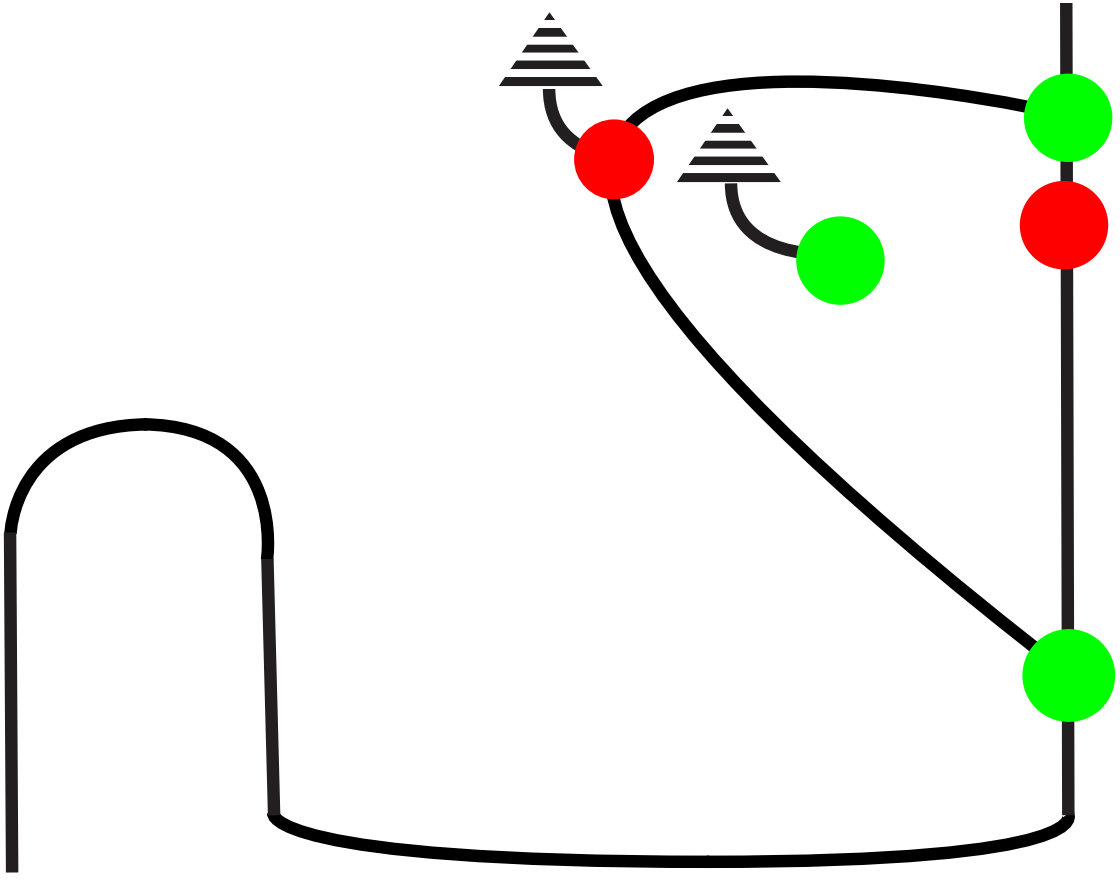
MEASUREMENT & CLASSICAL CONTROL

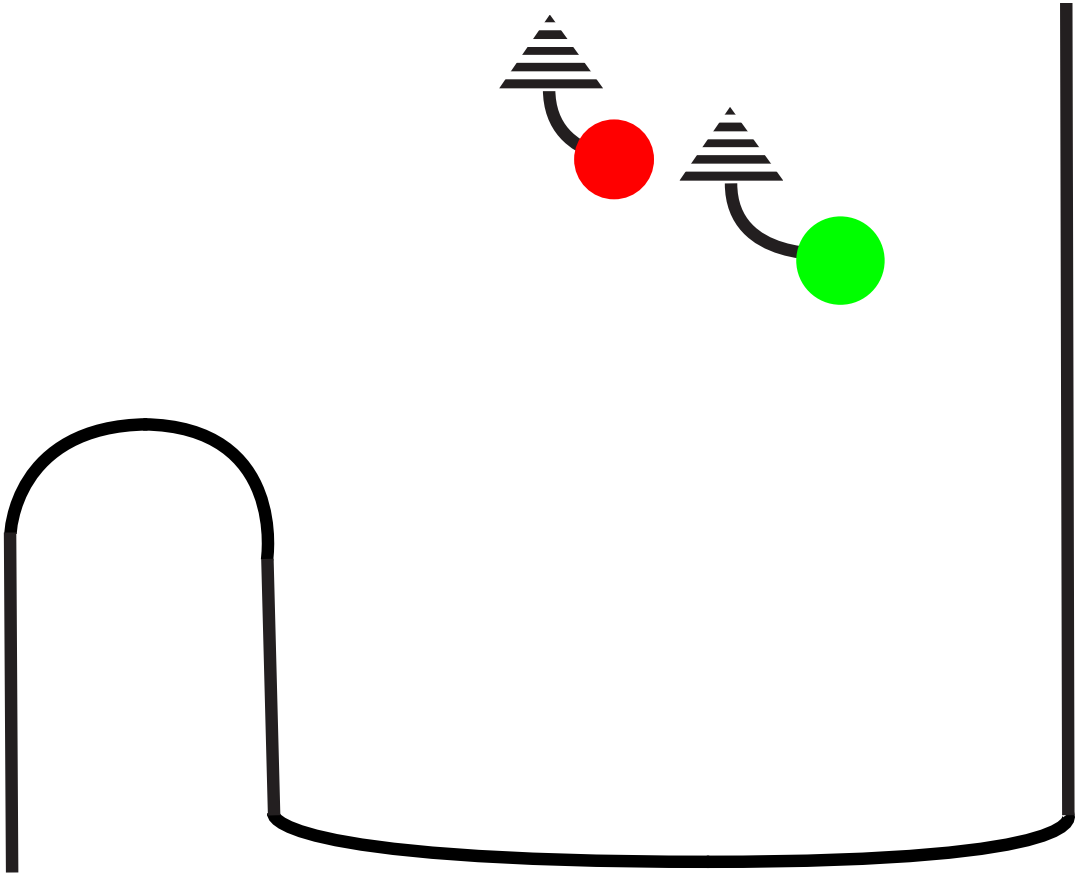


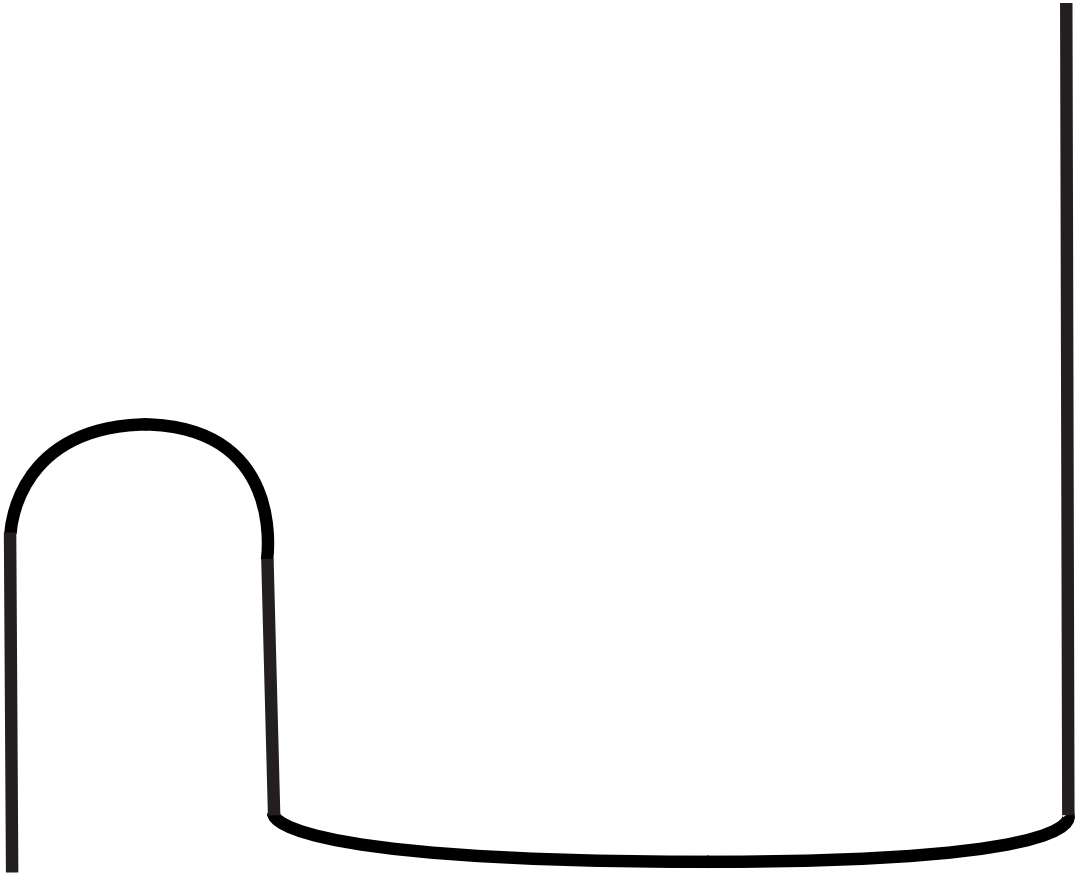




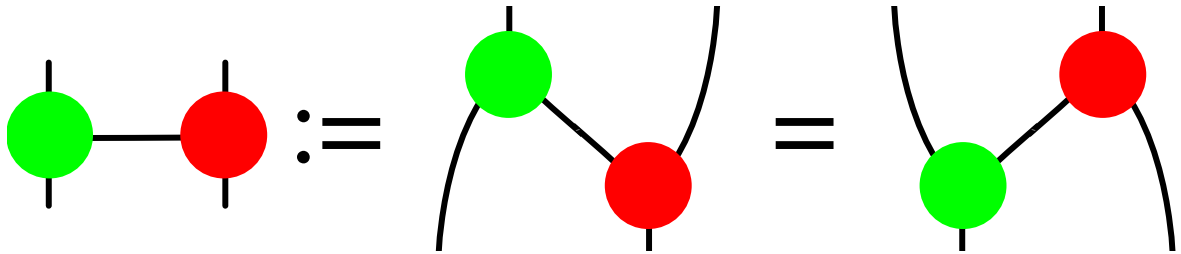








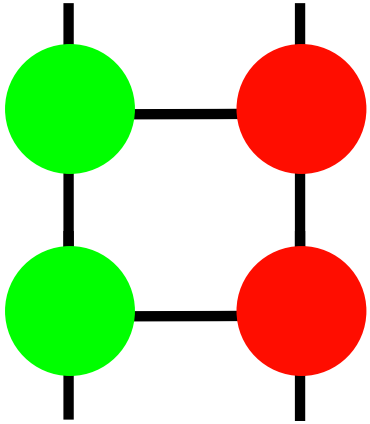
QUANTUM CIRCUITS

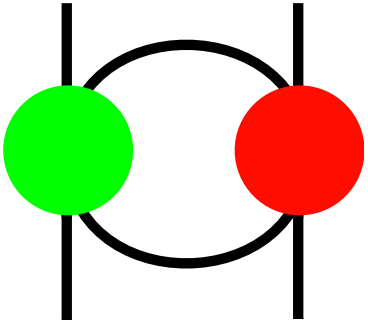


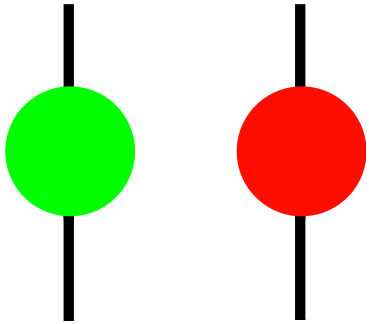
i.e.

$$(\delta_Z^\dagger \otimes 1) \circ (1 \otimes \delta_X) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = CNOT$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \circ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = ?$$

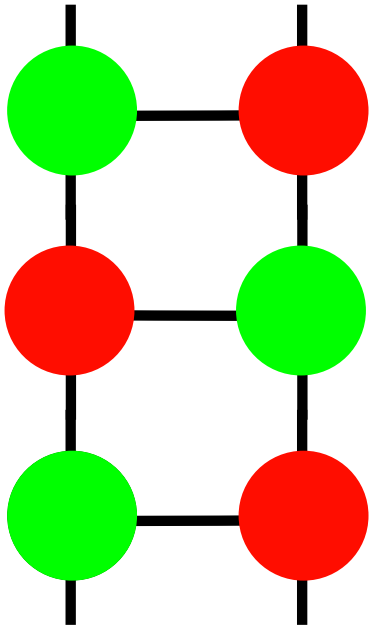


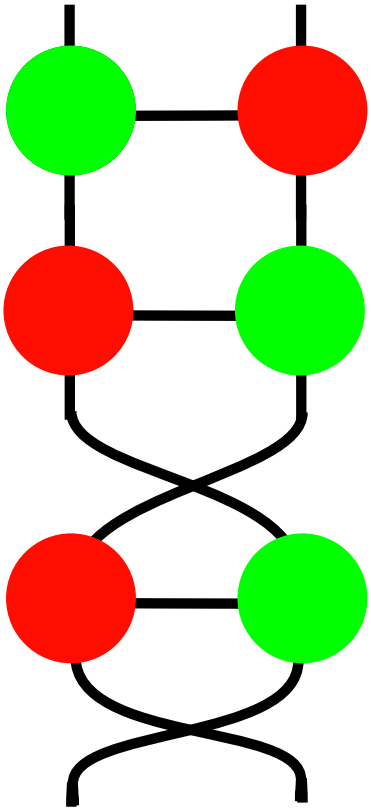


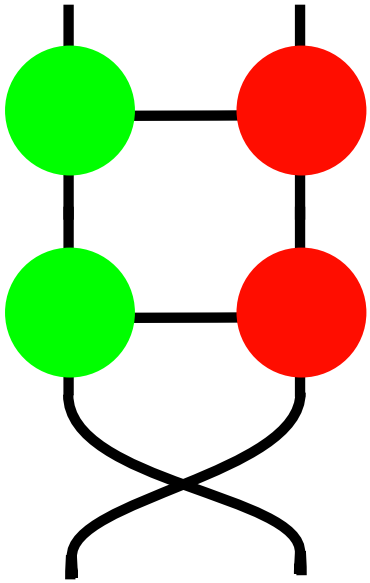


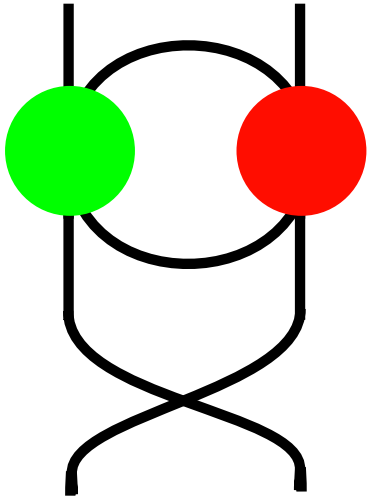
STRONG UNBIASEDNESS

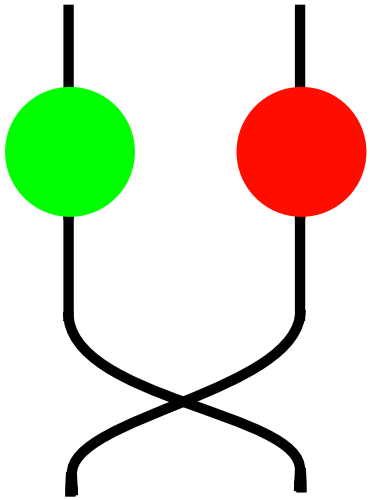
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \circ \sigma \circ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \circ \sigma \circ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = ?$$

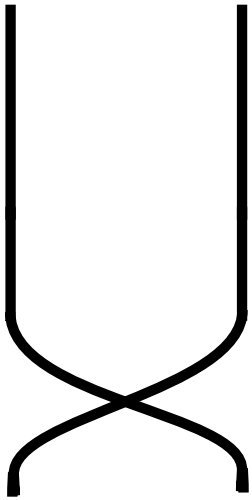




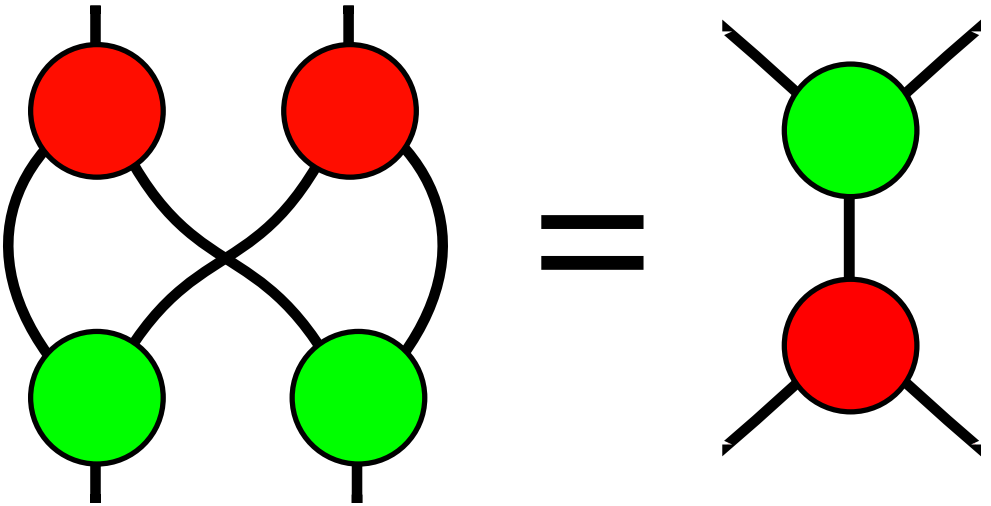








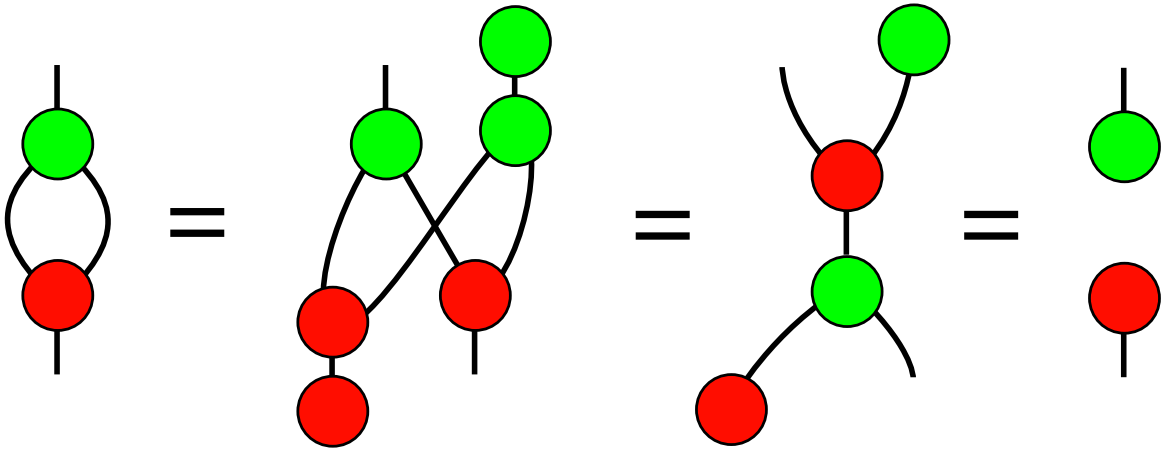
Def. Strong unbiasedness $\Leftrightarrow$



+ comults copy units of other colour.

BC & Ross Duncan (2008) *Interacting quantum observables*. ICALP'08 & New Journal of Physics **13**, 043016. arXiv:0906.4725

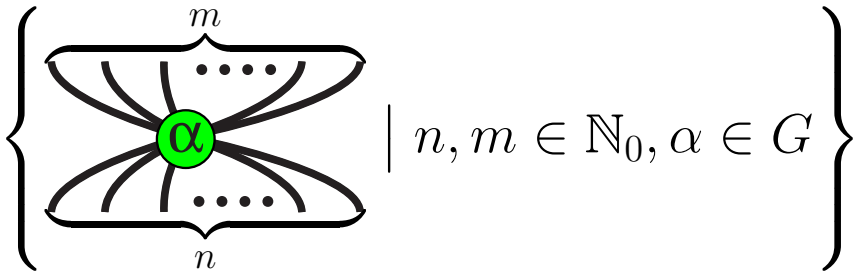
Strong complementarity $\Rightarrow$ complementarity

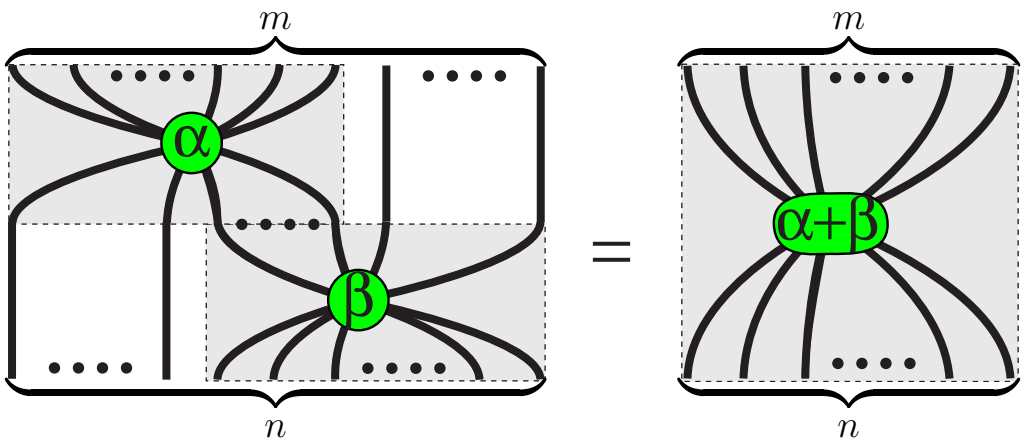


PHASES:

“decorated spiders”

Thm. ‘Unbiased states’ always form an Abelian group for spider multiplication with conjugate as inverse.



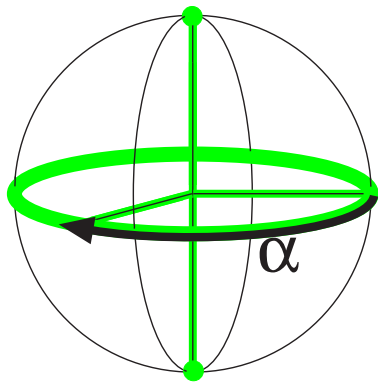


For qubits in **FHilb** with green $\equiv \{|0\rangle, |1\rangle\} \equiv Z$:

$$= \begin{pmatrix} 1 \\ e^{i\alpha} \end{pmatrix}_Z$$

$$= Z_\alpha = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{pmatrix}_Z$$

These are relative phases for Z, hence in X-Y:

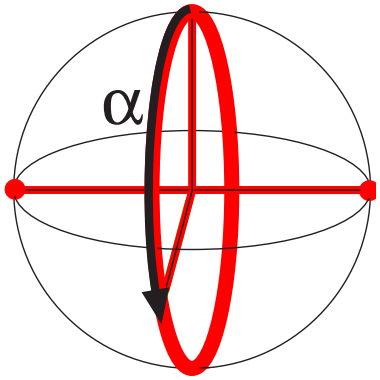


For qubits in **FHilb** with red $\equiv \{|+\rangle, |-\rangle\} \equiv X$:

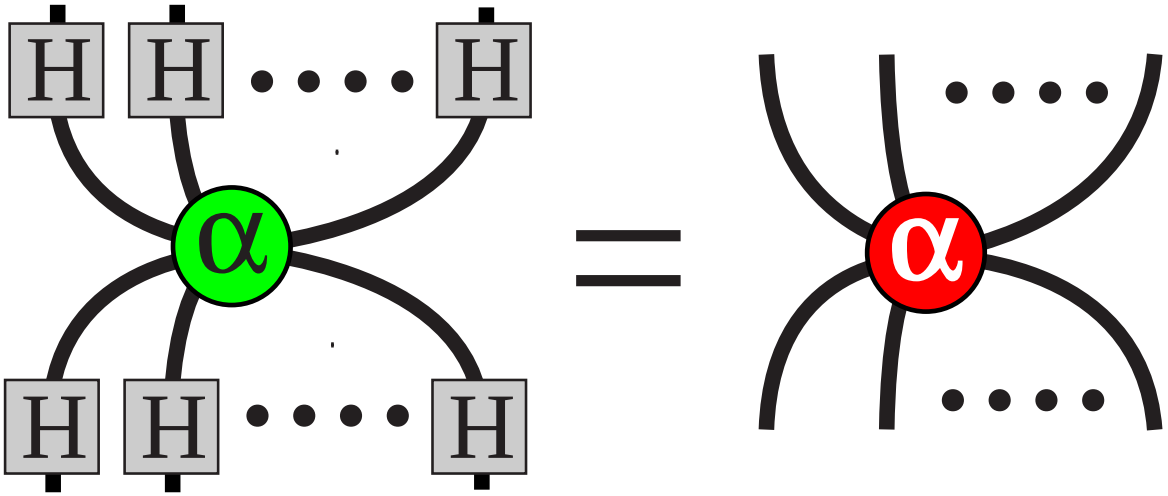
$$= \begin{pmatrix} 1 \\ e^{i\alpha} \end{pmatrix}_X$$

$$= X_\alpha = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{pmatrix}_X$$

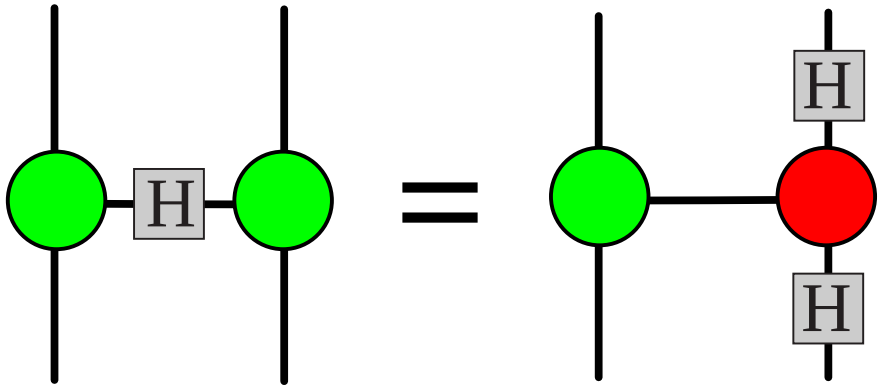
These are relative phases for X, hence in Z-Y:



— *colour changer* —

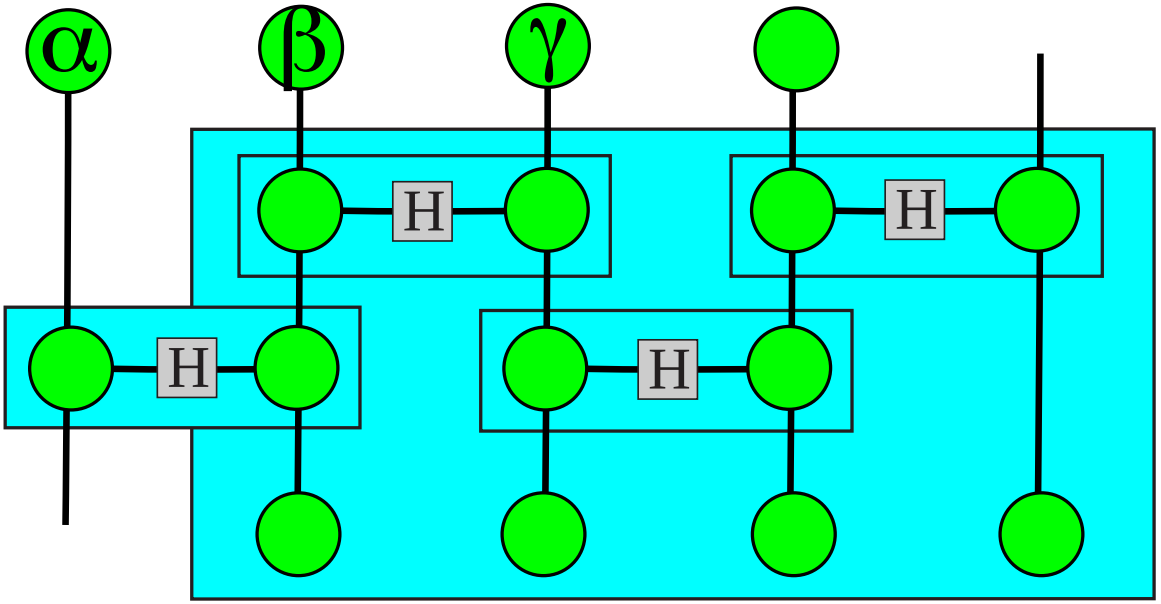


— *one* CZ *gate* —

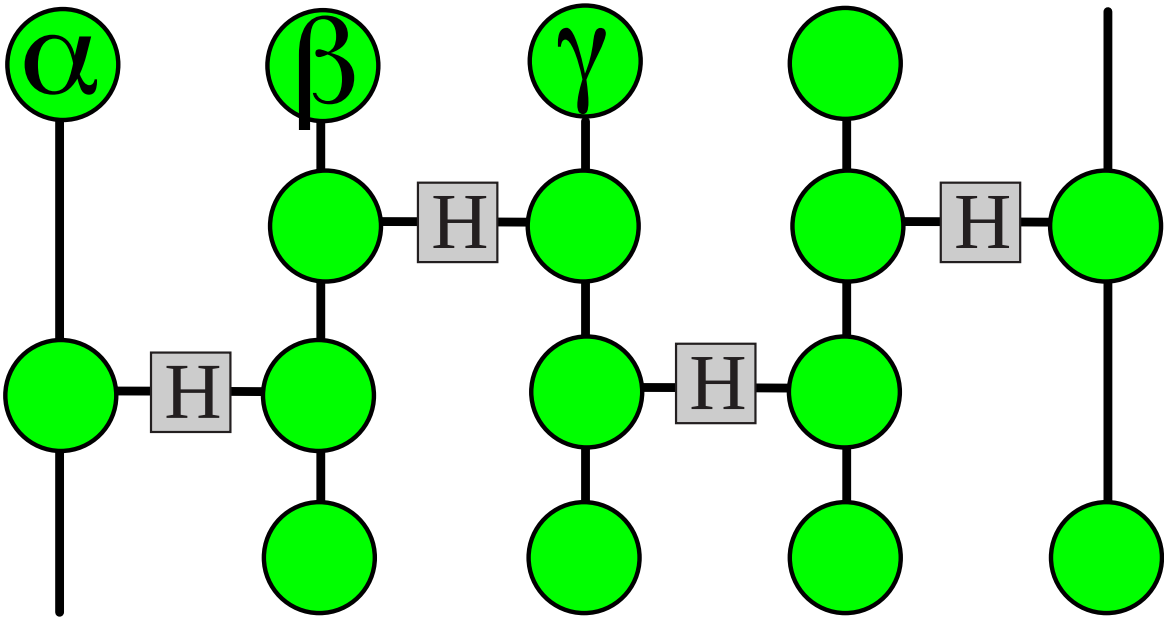


$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

— *MBQC is universal for qubit gates* —



— *MBQC is universal for qubit gates* —



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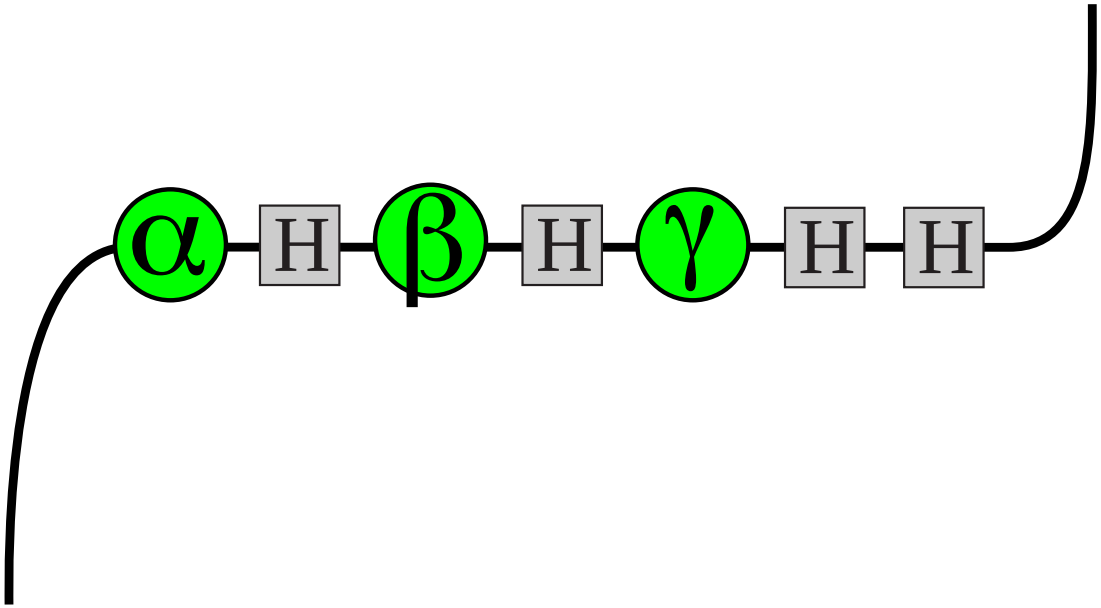

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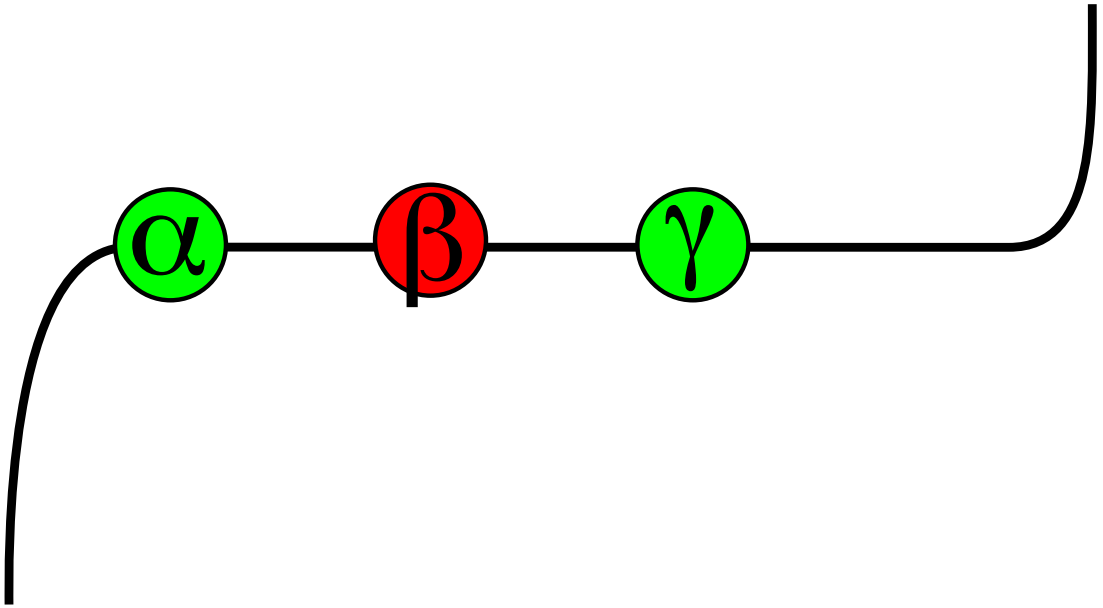


$\Rightarrow$ **Arbitrary one-qubit unitary**

— *universality* —

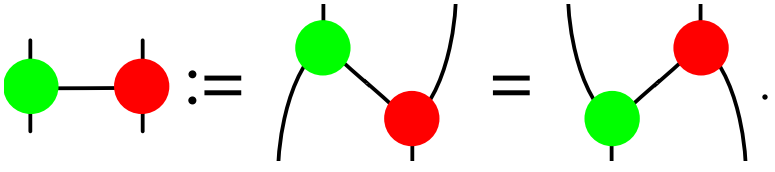
Thm. (BC & Ross Duncan) The above described graphical language, is **universal** for computing with qubits.

Proof. CNOT-gate + arbitrary one-qubit unitaries via their Euler angle decomposition yield all unitaries:

$$\Lambda^Z(\gamma) \circ \Lambda^X(\beta) \circ \Lambda^Z(\alpha) =$$

.

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} =$$

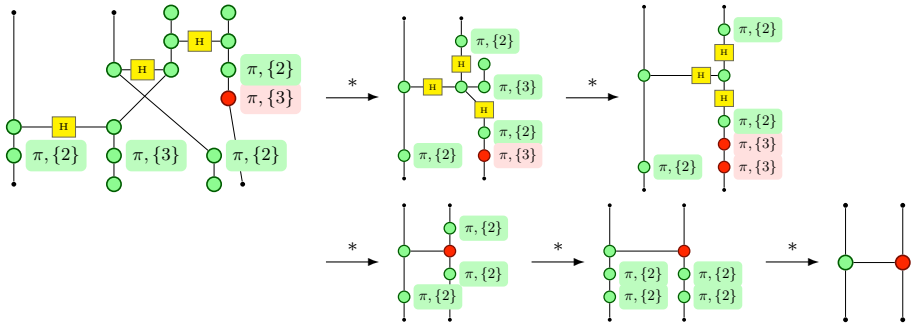


.

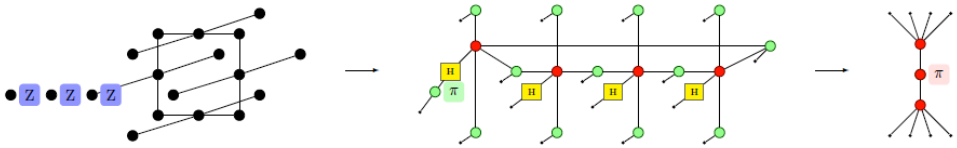
— *completeness II* —

Thm. (Miriam Backens, about a year ago) The above described graphical calculus with phases restricted to $\frac{\pi}{2}$-multiples, is **complete** for qubit stabiliser fragment of quantum computing, provided that we add the Euler angle decomposition of the Hadamard gate.

— *applications to QC models* —



Ross Duncan & Simon Perdrix (2010) *Rewriting measurement-based quantum computations with generalised flow*. ICALP'10. Survey: arXiv:1203.6242



Clare Horsman (2011) *Quantum picturalism for topological cluster-state computing*. New Journal of Physics **13**, 095011. arXiv:1101.4722.

Sergio Boixo & Chris Heunen (2012) *Entangled and sequential quantum protocols with dephasing*. **PRL 108**, 120402. arXiv:1108.3569

— *applications to quantum foundations* —

Toy qubits vs. true quantum theory in one language:

$$\frac{\text{Spekkens' qubit QM}}{\text{stabilizer qubit QM}} = \frac{Z_2 \times Z_2}{Z_4} = \frac{\text{local}}{\text{non-local}}$$

Coecke, Bill Edwards & Robert W. Spekkens (2010) Phase groups and the origin of non-locality for qubits. **QPL'10** `arXiv:1003.5005`

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Generalized Mermin arg. $\Leftrightarrow$ strong complementarity

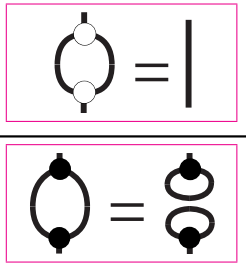
Coecke, Ross Duncan, Aleks Kissinger & Quanlong Wang (2012) *Strong complementarity and non-locality in categorical quantum mechanics*. **LiCS'12**. `arXiv:1203.4988`

— *multipartite entanglement structure* —

Tripartite SLOCC-classes as comm. Frobenius algs:

$$\frac{GHZ = |000\rangle + |111\rangle}{W = |001\rangle + |010\rangle + |100\rangle} = \frac{\text{‘special’ CFAs}}{\text{‘anti-special’ CFAs}}$$

$$= \frac{\boxed{}}{\boxed{}}$$

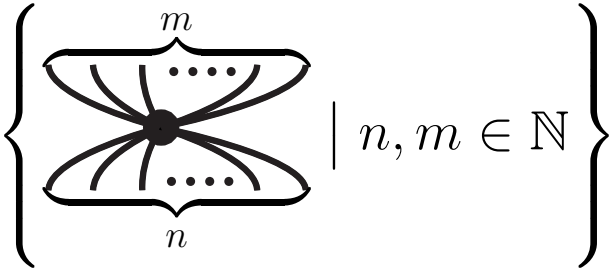


$$= \frac{\times}{+} \quad \Rightarrow \text{distributivity}$$

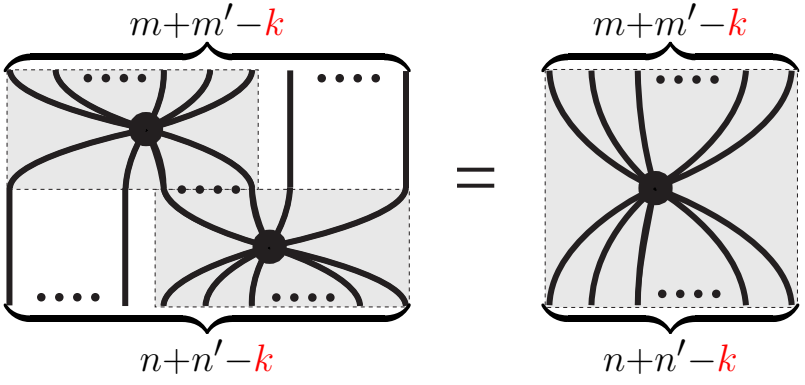
Coecke & Aleks Kissinger (2010) The compositional structure of multipartite quantum entanglement. ICALP'10. `arXiv:1002.2540`

— *GHZ-spiders* —

Data:

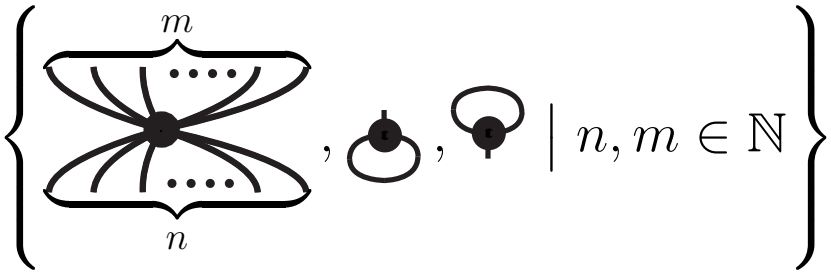


Rules:

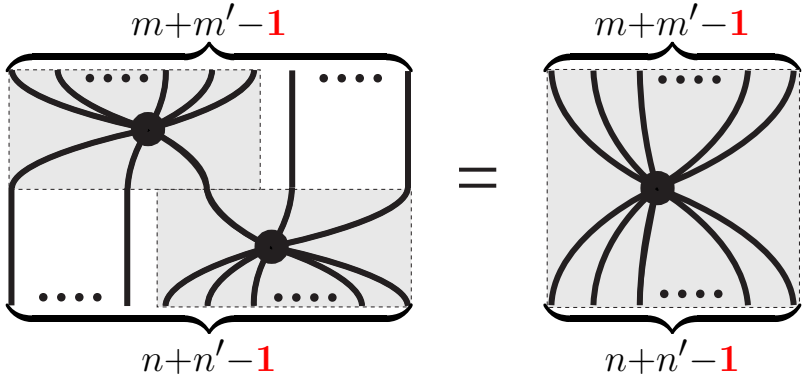


— *W-spiders* —

Data:

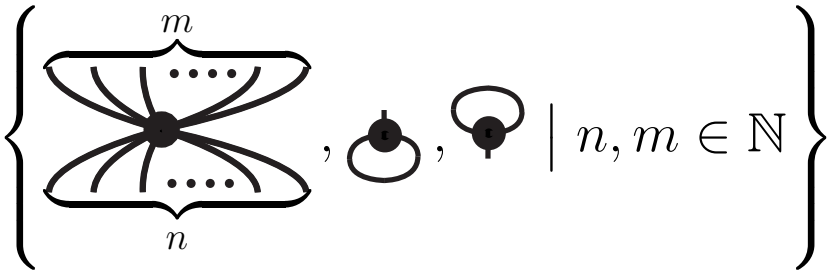


Rules:



— *W-spiders* —

Data:



Rules: