Structural reason for monogamy (and locality of certain macroscopic correlations)

Rui Soares Barbosa

Quantum Group
Computing Laboratory
University of Oxford

rui.soares.barbosa@cs.ox.ac.uk

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Introduction

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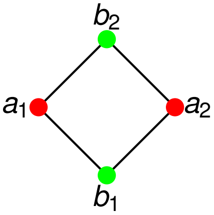
- Macroscopic correlations arising from microscopic models (Ramanathan et al. 2011: QM models) (only expectation values!)
- Use the general framework of Abramsky and Brandenburger (2011) and provide a structural reason using Vorob'ev's result (1962).
- Today, we will look only at a very simple example.

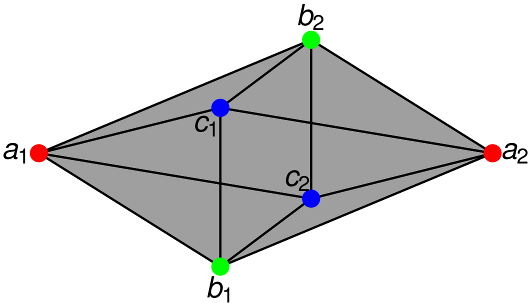
The setting

Measurement Scenarios

Abramsky-Brandenburger framework

- a finite set of measurements X;
- a cover $\mathcal{U}$ of X (or an abstract simplicial complex Σ on X), indicating the **compatibility** of measurements.





Examples: Bell-type scenarios, KS configurations, and more.

Empirical models

a family $(p_C)_{C \in \mathcal{U}}$, where p_C is a probability distribution on the outcomes of measurements in context C.

E.g. Z and X measurements on the W state:

	000	001	010	011	100	101	110	111
$a_1 b_1 c_1$	9	1	1	1	1	1	1	9
$a_1 b_1 c_2$	8	2	0	2	0	2	8	2
$a_1 b_2 c_1$	8	0	2	2	0	8	2	2
$a_1 b_2 c_2$	4	4	4	0	4	4	4	0
$a_2 b_1 c_1$	8	0	0	8	2	2	2	2
$a_2 b_1 c_2$	4	4	4	4	4	0	4	0
$a_2 b_2 c_1$	4	4	4	4	4	4	0	0
$a_2 b_2 c_2$	0	8	8	0	8	0	0	0

(every entry should be divided by 24)

The no-signalling condition

- Suppose Alice and Bob are space-like separated;
- Alice chooses to measure a_1; Bob can choose b_1 or b_2.
- What is $p(x \mid a_1)$ (prob of Alice obtaining the outcome x)?

$$p(x \mid a_1, b_1) := \sum_y p(x, y \mid a_1, b_1)$$

$$p(x \mid a_2, b_2) := \sum_y p(x, y \mid a_1, b_2)$$

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- Relativity implies that her measurement statistics cannot depend on Bob's choice of measurement:

$$p(x \mid a_1, b_1) = p(x \mid a_1, b_2)$$

I.e. it makes sense to speak of $p(x \mid a_1)$.

The no-signalling condition

In general, we require that our empirical models $(p_C)_{C \in \mathcal{U}}$ satisfy a compatibility condition:

p_C and $p_{C'}$ marginalise to the same distribution on the outcomes of measurements in $C \cap C'$.

For Bell-type multipartite scenarios, this condition corresponds to the usual **no-signalling**.

Non-locality and Contextuality

We are interested on whether a given empirical model admits a **local/non-contextual hidden variable** explanation (in the sense of Bell's theorem).

This is equivalent to the existence of a **global distribution** p_X (i.e. for all measurements at the same time) that marginalises to all p_C. (Abramsky, Brandenburger 2011).

Obstructions to such extensions are witnessed by **general Bell inequalities.** E.g. in bipartite scenario:

$$\sum_{i,j,x,y} \alpha(i,j,x,y)p(x,y \,|\, a_i, b_j) \leq R$$

Vorob'ev’s theorem

For which measurement compatibility structures $\mathcal{U}$ (or Σ) is it so that **any** no-signalling empirical model admits a global extension, i.e. is local/non-contextual?

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Vorob'ev (1962) derived a **necessay and sufficient** combinatorial condition on Σ for this to be the case. The idea is that such a scenario can be constructed by adding a measurement at a time in such a way that the new measurement belongs to only one maximal context.

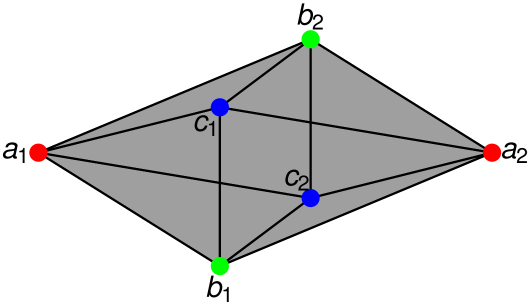
Monogamy

Tripartite example

Consider a tripartite scenario:

$$X = \{a_1, a_2, b_1, b_2, c_1, c_2\}$$

$$\mathcal{U} = \{\{a_i, b_j, c_k\} \mid i, j, k \in \{1, 2\}\}$$



Tripartite example

- Empirical model: no signalling probabilities

$$p(x, y, z \mid a_i, b_j, c_k)$$

where x, y, z are possible outcomes.

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- Consider the subsystem composed of A and B only, given by marginalisation (in QM, partial trace):

$$p(x, y \mid a_i, b_j) = \sum_z p(x, y, z \mid a_i, b_j, c_k)$$

(this is independent of c_k due to no-signalling).

Similarly define $p(x, z \mid a_i, c_k)$. (A and C)

Tripartite example

- Ramanathan et al.: A **macroscopic scenario** is obtained from an underlying microscopic scenario by **lumping together** certain measurements (e.g. spins in a given direction of several particles give rise to a magnetisation measurement in that direction). The merged measurements must be 'symmetric' in some sense.
- Consider B and C to be in the same 'macroscopic' site. The symmetry identifies the measurements $b_1 \sim c_1$ and $b_2 \sim c_2$, giving rise to macroscopic measurements m_1 and m_2.

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- They consider emergent 'macroscopic' probabilities given by an **average**:

$$p(a_i, m_j = x, y) = \frac{1}{2}\Big(p(x, y \mid a_i, b_j) + p(x, y \mid a_i, c_j) \Big)$$

Monogamy and locality of quotient model

Consider any **(general) Bell inequality** for a bipartite scenario: a set of coefficients $\alpha(i, j, x, y)$ and a bound R.

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$$\Leftrightarrow$$

$$\sum_{i,j,x,y} \frac{1}{2} \alpha(i,j,x,y) \left(p(x,y \mid a_i, b_j) + p(x,y \mid a_i, c_j) \right) \leq R$$

$$\Leftrightarrow$$

$$\sum_{i,j,x,y} \alpha(i,j,x,y) p(x,y \mid a_i, b_j) + \sum_{i,j,x,y} \alpha(i,j,x,y) p(x,y \mid a_i, c_j) \leq 2R$$

Monogamy and locality of quotient model

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$$\Leftrightarrow$$

$$\sum_{i,j,x,y} \alpha(i, j, x, y) p(x, y \mid a_i, b_j) + \sum_{i,j,x,y} \alpha(i, j, x, y) p(x, y \mid a_i, c_j) \leq 2R$$

The quotient model $p(a_i, m_j = \cdots)$ **satisfies the inequality** if and only if Alice in the microscopic model is **monogamous** with respect to violating it.

Example: W-state

	00	01	10	11
$a_1 m_1$	10	2	2	10
$a_1 m_2$	8	4	8	4
$a_2 m_1$	8	8	4	4
$a_2 m_2$	8	8	8	0

(every entry should be divided by 24)

This is **local**! This is general for any empirical model.

Another example model

	000	001	010	011	100	101	110	111
$a_1 b_1 c_1$	1	1	0	0	0	0	1	1
$a_1 b_1 c_2$	1	1	0	0	0	0	1	1
$a_1 b_2 c_1$	1	1	0	0	0	0	1	1
$a_1 b_2 c_2$	1	1	0	0	0	0	1	1
$a_2 b_1 c_1$	1	1	0	0	0	0	1	1
$a_2 b_1 c_2$	1	1	0	0	0	0	1	1
$a_2 b_2 c_1$	0	0	1	1	1	1	0	0
$a_2 b_2 c_2$	0	0	1	1	1	1	0	0

(every entry should be divided by 4)

Another example model

	00	01	10	11
$a_1 b_1$	2	0	0	2
$a_1 b_2$	2	0	0	2
$a_2 b_1$	2	0	0	2
$a_2 b_2$	0	2	2	0

(divided by 4)

	00	01	10	11
$a_1 c_1$	1	1	1	1
$a_1 c_2$	1	1	1	1
$a_2 c_1$	1	1	1	1
$a_2 c_2$	1	1	1	1

(divided by 4)

left: maximally non-local, right: local

	00	01	10	11
$a_1 m_1$	3	1	1	3
$a_1 m_1$	3	1	1	3
$a_1 m_1$	3	1	1	3
$a_1 m_1$	1	3	3	1

(every entry should be divided by 8)

Again, this is **local**!

Structural Reason

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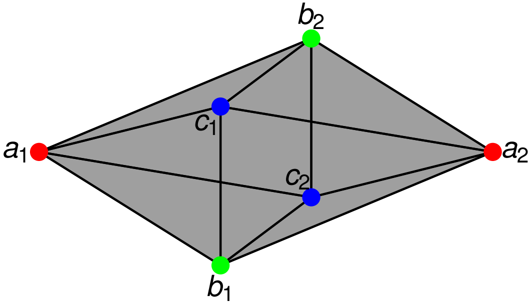

- Measurement scenario: simplicial complex $\mathfrak{D}_2 * \mathfrak{D}_2 * \mathfrak{D}_2$.
- We identify B and C: $b_1 \sim c_1$, $b_2 \sim c_2$.
- The macro scenario arises as a quotient.

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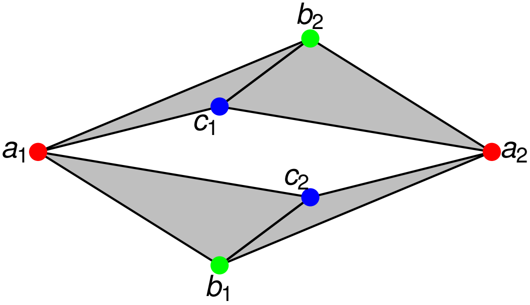

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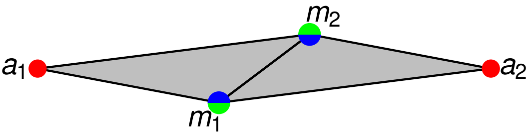

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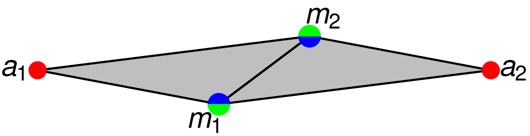

- This quotient complex satisfies the **Vorob'ev condition.**
- Therefore, no matter which micro model $p(a_i, b_j, c_k = \cdots)$ we start from, the averaged macro correlations $p(a_i, m_j = \cdots)$ are local!
- In particular, they satisfy any Bell inequality. Hence, the original tripartite model also satisfies a **monogamy relation** for any Bell inequality.

Summary/Conclusions

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- A model satisfies a **monogamy** relation for a Bell inequality iff its the emergent averaged correlations (quotient model) satisfy the Bell inequality.
- So, if the quotient scenario is Vorob'ev-regular, then **any no-signalling empirical model** is monogamous wrt to all Bell inequalities (since the emergent averaged correlations are local/non-contextual).

Summary/Conclusions

- In particular, we proved that this is the case for multipartite Bell-type scenarios provided the number of parties being identified as belonging to each 'macro' site is larger than the number of measurement settings available to each of them.
- Our approach highlights the **reason why** monogamy relations for general multipartite Bell inequalities follow from no-signalling alone, generalising the result of Pawłowski and Brukner (2009) from bipartite to multipartite. (It also shows that what Ramanathan et al. proved holds not only for QM but for any no-signalling theory.)
- The approach is not restricted to multipartite Bell-type scenarios. More generally, we can apply the same ideas to derive monogamy relations for contextuality inequalities.

Questions...

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