Physics on the boundary between Quantum Mechanics and Gravity

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Quantum Fields, Gravity and Information
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Why introduce something new into Quantum mechanics?

- Schrödinger Cat States
- Description of measurement process
- Linearity of quantum mechanics vs Nonlinearity of General Relativity

Measurement in quantum mechanics

Density Matrix

$$\rho \text{ before measurement } \rightarrow \frac{P_n \rho P_n}{Tr(P_n \rho P_n)} \text{ after measurement}$$

Measurement Problem

$$|s_n\rangle \otimes |A_1\rangle \rightarrow |s_n\rangle \otimes |A_n\rangle$$

Problem:

$$\frac{1}{\sqrt{2}}(|s_n\rangle + |s_l\rangle) \rightarrow \frac{1}{\sqrt{2}}(|s_n\rangle \otimes |A_n\rangle + |s_l\rangle \otimes |A_l\rangle)$$

Entanglement between Apparatus and measured system.

Interpretation:

Measuring either $|s_n\rangle \otimes |A_n\rangle$ or $|s_l\rangle \otimes |A_l\rangle$ with probability $\frac{1}{2}$.

This leads to two different time evolutions:

- Schrödinger evolution $\rightarrow$ linear, deterministic
- Wavepacket reduction $\rightarrow$ nonlinear, stochastic

Possible ways to deal with this problem[Bassi,2003]:

- Incompleteness
- formal Completeness
- with different Individuals
- with identical Individuals
- Two dynamical principles
- Unifyied dynamics

Dynamical reduction models:
Simple example:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|x_1\rangle + |x_2\rangle)$$

$$|\psi\rangle\langle\psi| = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

Look for "collapsed" system

$$\frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \rightarrow \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Also evolution has to give the right mixture, i.e. individual parts have to have definite macroscopic properties.

Linear, stochastic

$$i\hbar\partial_t|\psi(t)\rangle = \sum_i P_i V_i(t)|\psi(t)\rangle$$

$$\langle\langle V_i(t)\rangle\rangle = 0$$

$$\langle\langle V_i(t)V_j(t)\rangle\rangle = \gamma\delta_{ij}\delta(t-t')$$

$$\partial_t\langle x_i|\rho(t)|x_j\rangle = \gamma(\delta_{ij}-1)\langle x_i|\rho(t)|x_j\rangle$$

BUT

$$\langle\psi(t)|P_i|\psi(t)\rangle = \langle\psi(0)|P_i|\psi(0)\rangle$$

Nonlinear, deterministic

Ensembles E_1, E_2

$$E_1\ \{a_i, |\phi_i\rangle\} \quad E_2\ \{b_i, |\varphi_i\rangle\}$$

Purification

$$\sum_i a_i|\phi_i\rangle\otimes|\eta_i\rangle = \sum_i b_i|\varphi_i\rangle\otimes|\zeta_i\rangle$$

Two regions $X_1 \to \phi, \varphi, X_2 \to \eta, \zeta$
Create ensembles by measuring in X_2. Due to No-Signalling $\to$ only linear operations allowed![Gisin,1989]

Collapse Models are Nonlinear and Stochastic.

Bassi, Dürr, Hinrichs 2013:[Bassi,2013]

"Collapse models are the only possible
nonlinear extensions of the Schrödinger equation,
compatible with the no-faster-than-light assumption."

Only equations of type

$$d\psi_t = \left(-iHdt + \sum_{k=1}^{n} (L_k - l_{k,t}) dW_{k,t} - \frac{1}{2} \sum_{k=1}^{n} (L_k^\dagger L_k + 2 l_{k,t} L_k + |l_{k,t}|^2) dt \right) \psi_t$$

$$l_{k,t} = \frac{1}{2} \langle \psi_t | (L_k^\dagger + L_k) | \psi_t \rangle$$

are allowed.

The CSL Model

$$\partial_t \rho(t) = -\frac{i}{\hbar}[H, \rho(t)] + \gamma \int d^3x N(x)\rho(t)N(x) - \frac{\gamma}{2}\int d^3x \{N^2(x), \rho(t)\}$$

where $N(x)$ denotes the preferred basis

$$N(x) = \int d^3y g(y-x)a^\dagger(y)a(y) \qquad g(x) = \left(\frac{\alpha}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{\alpha}{2}(x)^2}$$

Simple model:

$$\partial_t \rho(t) = \gamma \left(\frac{\alpha}{4\pi}\right)^{\frac{3}{2}} \sum_k \left[\sum_i N_i^{(k)}\rho(t)N_i^{(k)} - \frac{1}{2}\sum_i \{N_i^{(k)2}, \rho(t)\}\right]$$

$$\langle n_1^{(k)}, \ldots |\rho(t)| m_1^{(k)}, \ldots\rangle = e^{-\frac{\gamma}{2}\left(\frac{\alpha}{4\pi}\right)^{\frac{3}{2}} \sum_{i,k}(n_i^{(k)} - m_i^{(k)})} \langle n_1^{(k)}, \ldots |\rho(0)| m_1^{(k)}, \ldots\rangle$$

Question:

Meaning of Parameters?

Replace Number operator with Mass Density operator

$$M(x) = \sum_k m_k N_k(x)$$

$$\partial_t \rho(t) = -\frac{i}{\hbar}[H, \rho(t)] + \frac{\gamma}{m_0^2}\int d^3x M(x)\rho(t)M(x) - \frac{\gamma}{2m_0^2}\int d^3x \{M^2(x), \rho(t)\}$$

Thus relate collapse to the mass density of the particles. Still two arbitrary parameters needed.

One step further:

Diosi's proposition of collapse due to gravity.
(QMUDL: quantum mechanics with universal density localization)

Introduce mass density

$$f(x) = \frac{M}{V}\theta(R - |x' - x|)$$

$$\partial_t \rho(t) = -\frac{i}{\hbar}[H, \rho(t)] - \frac{G}{2\hbar}\int\int \frac{d^3x_1 d^3x_2}{|x_1 - x_2|}[f(x_1), [f(x_2), \rho(t)]]$$

for description of decoherence processes due to gravity.

Assume Gravitational Potential as cause for Collapse

$$U(x) = -G \int \frac{f(x_1)f(x_2)}{|x_1 - x_2|} dx_1 dx_2$$

This leads to

$$\langle x'|\rho(t)|x''\rangle = e^{\Gamma(|x'-x''|)t}\langle x'|\rho(0)|x''\rangle$$

where

$$\Gamma(|x'-x''|) = \frac{1}{\hbar}(U(0) - U(|x'-x''|))$$

Assume Gravitational Potential as cause for Localization

Modify Schrödinger Equation:

$$i\hbar\partial_t\psi = \left(-\frac{\hbar^2}{2m}\Delta - Gm^2\int\frac{|\psi^2|}{|x-y|}d^3y\right)\psi$$

$$\Delta\Phi = 4\pi Gm|\psi|^2$$

Nonlinear Integro-Differential Equation!

Scaling of Schrödinger Newton

$$m_\mu = \mu m$$

$$t_\mu = \mu^5 t$$

$$r_\mu = \mu^3 r$$

$$\psi_\mu = \mu^{\frac{9}{2}} \psi$$

Numerical solutions: Solitons!

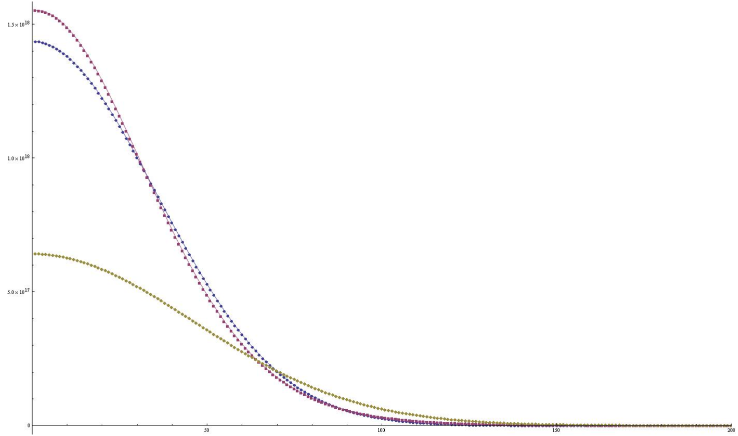

Abbildung : Solitonic solution of the Schrödinger Newton equation. Stepsize: $r = 10^{-8}m$, Blue: Gauss Packet at $t = 0$, Red: Schrödinger Newton solution after $20000s$, Yellow: Schrödinger solution after $20000s$.

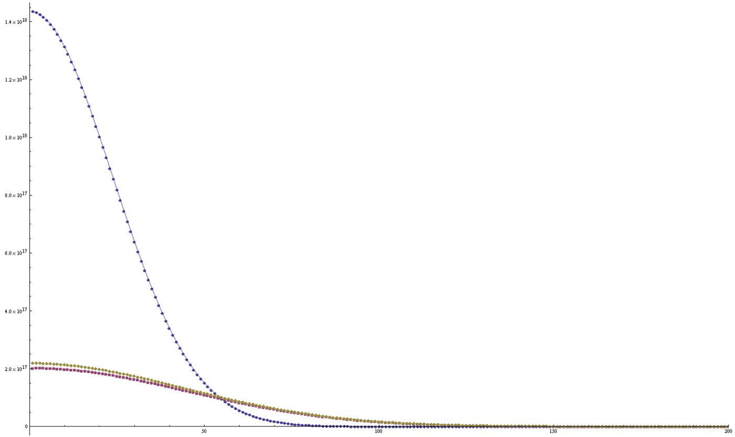

Abbildung : Solution of the Schrödinger Newton equation. Stepsize: $r = 10^{-14}m$, Blue: Gauss Packet at $t = 0$, Red: Schrödinger Newton solution after $10^{-10}s$, Yellow: Schrödinger solution after $10^{-10}s$.

Conclusions

- Framework beyond quantum mechanics.
- Is the only feasible Ansatz that does not contradict the No-faster-than-light assumption.
- Allows explanation of the ad-hoc collapse of the wavefunction without introducing a second time evolution.
- Introduced parameters may be linked to Gravity.
- Experimental tests may be possible to distinguish these models from standard quantum mechanics.

Thank you for your attention!

References:

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