Wavepacket detection with the Unruh-DeWitt model

E. Martín-Martínez M. Montero M. del Rey

Universidad Autónoma de Madrid & Instituto de Física Fundamental, CSIC

Quantum Fields, Gravity and Information
University of Nottingham, April 2013

- *Wavepacket detection with the Unruh-DeWitt model*, E. Martín-Martínez, M. Montero and M. del Rey, Phys. Rev. D 87, 064038 (2013).


- Edu, Miguel, Marco $\in$ QUINFOG $\subset$ IFF

Two-level systems

- Simplest quantum system: Two-level $\mathcal{H} = \{|0\rangle, |1\rangle\}$.

$$H = \frac{\hbar\Omega}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{\hbar\Omega}{2}\sigma_z$$

- As a detector model: $|0\rangle =$ no click, $|1\rangle =$ click.
- Mapping to $\mathfrak{u}(2) \Rightarrow$ Any operator may be decomposed as

$$\mathcal{O} = \alpha\mathbf{I} + \beta\sigma_z + \gamma\sigma_x + \delta\sigma_y.$$

Two-level systems

- Simplest quantum system: Two-level $\mathcal{H} = \{|0\rangle, |1\rangle\}$.

$$H = \frac{\hbar\Omega}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{\hbar\Omega}{2}\sigma_z$$

- As a detector model: $|0\rangle =$ no click, $|1\rangle =$ click.
- Mapping to $\mathfrak{u}(2) \Rightarrow$ Any operator may be decomposed as

$$\mathcal{O} = \alpha\mathbf{I} + \beta\sigma_z + \gamma\sigma_x + \delta\sigma_y.$$

Coupling to fields

Now add a scalar field $\phi(x)$.

- Turn on an interaction between the two-level system and $\phi(x)$... This is the **Unruh-DeWitt detector**

$$H_{\text{int.}} = g\phi(x_{\text{detector}})\sigma_x$$

- *Very* general model: We can extend to other kinds of fields by multiplying by an adequate current like j^a.

Coupling to fields

Now add a scalar field $\phi(x)$.

- Turn on an interaction between the two-level system and $\phi(x)$... This is the **Unruh-DeWitt detector**

$$H_{\text{int.}} = g\phi(x_{\text{detector}})\sigma_x$$

- *Very* general model: We can extend to other kinds of fields by multiplying by an adequate current like j^a.

Coupling to fields

Now add a scalar field $\phi(x)$.

- Turn on an interaction between the two-level system and $\phi(x)$... This is the **Unruh-DeWitt detector**

$$H_{\text{int.}} = g\phi(x_{\text{detector}})\sigma_x$$

- *Very* general model: We can extend to other kinds of fields by multiplying by an adequate current like j^a.

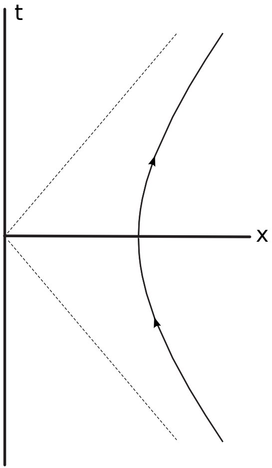
An example: The Unruh effect

- $x_{\text{detector}} = x(\tau) = \frac{c^2}{a}\cosh\left(\frac{a\tau}{c}\right)$.
- Transition prob. to first order perturbation theory:

$$\Gamma = \frac{g^2\hbar\Omega}{2\pi\hbar^2}\frac{1}{e^{\hbar\Omega/kT_U}-1},$$

with $T_U = \hbar a/(2\pi c k_{\text{B}})$ the Unruh temperature.

- Lots of applications: James-Cummings, collective excitations of Bose crystals...



UdW: Smearing things out

- The Unruh-DeWitt detector,

$$H_{\text{int.}} = g\phi(x_{\text{detector}})\sigma_x$$

is pointlike. $\to$ regularisation-dependent, divergences.

- *Smeared*[1] UdW model,

$$H_{\text{int.}}^{\text{smeared}} = g\sigma_x \int dx\, F(x)\phi(x)$$

- Or, being a bit more general,

$$H_{\text{int.}}^{\text{smeared}} = g \int dx \left(F(x)\sigma^+ + F^*(x)\sigma^- \right),$$

where $\sigma^\pm \equiv \sigma_x \pm i\sigma_y$.

[1] S. Schlicht, Classical and Quantum Gravity 21, 4647 (2004), J.Louko and A. Satz, Class.Quant.Grav. 23, 6321 (2006).

UdW: Smearing things out

- The Unruh-DeWitt detector,

$$H_{\text{int.}} = g\phi(x_{\text{detector}})\sigma_x$$

is pointlike. → regularisation-dependent, divergences.

- *Smeared*[1] UdW model,

$$H_{\text{int.}}^{\text{smeared}} = g\sigma_x \int dx\, F(x)\phi(x)$$

- Or, being a bit more general,

$$H_{\text{int.}}^{\text{smeared}} = g \int dx \left(F(x)\sigma^+ + F^*(x)\sigma^- \right),$$

where $\sigma^\pm \equiv \sigma_x \pm i\sigma_y$.

[1] S. Schlicht, Classical and Quantum Gravity 21, 4647 (2004), J.Louko and A. Satz, Class.Quant.Grav. 23, 6321 (2006).

UdW: Smearing things out

- The Unruh-DeWitt detector,

$$H_{\text{int.}} = g\phi(x_{\text{detector}})\sigma_x$$

is pointlike. $\rightarrow$ regularisation-dependent, divergences.

- *Smeared*[1] UdW model,

$$H_{\text{int.}}^{\text{smeared}} = g\sigma_x \int dx\, F(x)\phi(x)$$

- Or, being a bit more general,

$$H_{\text{int.}}^{\text{smeared}} = g \int dx \left(F(x)\sigma^+ + F^*(x)\sigma^- \right),$$

where $\sigma^\pm \equiv \sigma_x \pm i\sigma_y$.

[1] S. Schlicht, Classical and Quantum Gravity 21, 4647 (2004), J.Louko and A. Satz, Class.Quant.Grav. 23, 6321 (2006).

UdW: Smearing things out

- The Unruh-DeWitt detector,

$$H_{\text{int.}} = g\phi(x_{\text{detector}})\sigma_x$$

  is pointlike. $\rightarrow$ regularisation-dependent, divergences.
- *Smeared*[1] UdW model,

$$H_{\text{int.}}^{\text{smeared}} = g\sigma_x \int dx\, F(x)\phi(x)$$

- Or, being a bit more general,

$$H_{\text{int.}}^{\text{smeared}} = g \int dx \left(F(x)\sigma^+ + F^*(x)\sigma^-\right),$$

  where $\sigma^\pm \equiv \sigma_x \pm i\sigma_y$.

[1] S. Schlicht, Classical and Quantum Gravity 21, 4647 (2004), J.Louko and A. Satz, Class.Quant.Grav. 23, 6321 (2006).

UdW generalisation

- The UdW detector is supposed to model a physical system!

Which one?

- We choose an atom in an EM field (2 polarizations).

$$H_{\text{int.}} = g \sum_{\lambda=+,-} \int dx [F_\lambda(x)\sigma^+ + F_\lambda^*(x)\sigma^-] \cdot A_\lambda(x)$$

- Spin-1 version of UdW
- **Goal**: Get $F_\lambda(x)$ in terms of physical quantities.

UdW generalisation

- The UdW detector is supposed to model a physical system!

Which one?

- We choose an atom in an EM field (2 polarizations).

$$H_{\text{int.}} = g \sum_{\lambda=+,-} \int dx [F_\lambda(x)\sigma^+ + F^*_\lambda(x)\sigma^-] \cdot A_\lambda(x)$$

- Spin-1 version of UdW
- **Goal**: Get $F_\lambda(x)$ in terms of physical quantities.

UdW generalisation

- The UdW detector is supposed to model a physical system!

Which one?

- We choose an atom in an EM field (2 polarizations).

$$H_{\text{int.}} = g \sum_{\lambda=+,-} \int dx [F_\lambda(x)\sigma^+ + F^*_\lambda(x)\sigma^-] \cdot A_\lambda(x)$$

- Spin-1 version of UdW
- **Goal**: Get $F_\lambda(x)$ in terms of physical quantities.

UdW generalisation

- The UdW detector is supposed to model a physical system!

Which one?

- We choose an atom in an EM field (2 polarizations).

$$H_{\text{int.}} = g \sum_{\lambda=+,-} \int dx [F_\lambda(x)\sigma^+ + F^*_\lambda(x)\sigma^-] \cdot A_\lambda(x)$$

- Spin-1 version of UdW
- **Goal**: Get $F_\lambda(x)$ in terms of physical quantities.

QED interaction

- The QED interaction between an atom D and a weak external EM field is

$$H^{\text{QED}}_{\text{int.}} = e\boldsymbol{p}_{\text{D}} \cdot \boldsymbol{A}(\boldsymbol{x}, 0)$$
$$= e\boldsymbol{p}_{\text{D}} \cdot \sum_{\lambda=+,-} \int \frac{d\boldsymbol{p}}{\sqrt{2p}} \left[\epsilon_{\boldsymbol{p},\lambda} a^\dagger_{\boldsymbol{p},\lambda} e^{-ipx} + \epsilon^*_{\boldsymbol{p},\lambda} a_{\boldsymbol{p},\lambda} e^{ipx}\right].$$

- We work in Coulomb gauge $\nabla \cdot \boldsymbol{A} = 0$.

From QED to UdW

- To put it in a UdW-like form, restrict atomic dynamics to a two-level system,

$$\mathcal{H} \approx \{|\Psi_g\rangle, |\Psi_e\rangle\}, \quad \boldsymbol{p}_{\mathsf{D}} \approx \begin{pmatrix} \langle\Psi_g|\boldsymbol{p}_{\mathsf{D}}|\Psi_g\rangle & \langle\Psi_g|\boldsymbol{p}_{\mathsf{D}}|\Psi_e\rangle \\ \langle\Psi_e|\boldsymbol{p}_{\mathsf{D}}|\Psi_g\rangle & \langle\Psi_e|\boldsymbol{p}_{\mathsf{D}}|\Psi_e\rangle \end{pmatrix}.$$

- Then, expand $H_{\text{int.}}^{\text{QED}} = e\boldsymbol{p}_{\mathsf{D}} \cdot \boldsymbol{A}(\boldsymbol{x}, 0)$ in terms of Pauli matrices,

$$H_{\text{int.}}^{\text{QED}} = \alpha \mathbf{I} + \beta\sigma_z + \gamma\sigma_x + \delta\sigma_y.$$

  Here $\alpha, \beta, \gamma, \delta$ are **field operators.**
- Rearrange and forget about α, β terms to get back UdW Hamiltonian with **complex** profile

$$\boldsymbol{F}(\boldsymbol{x}) = -i\Psi_e^*(\boldsymbol{x})\nabla\Psi_g(\boldsymbol{x})$$

From QED to UdW

- To put it in a UdW-like form, restrict atomic dynamics to a two-level system,

$$\mathcal{H} \approx \{|\Psi_g\rangle, |\Psi_e\rangle\}, \quad \boldsymbol{p}_{\mathrm{D}} \approx \begin{pmatrix} \langle\Psi_g|\boldsymbol{p}_{\mathrm{D}}|\Psi_g\rangle & \langle\Psi_g|\boldsymbol{p}_{\mathrm{D}}|\Psi_e\rangle \\ \langle\Psi_e|\boldsymbol{p}_{\mathrm{D}}|\Psi_g\rangle & \langle\Psi_e|\boldsymbol{p}_{\mathrm{D}}|\Psi_e\rangle \end{pmatrix}.$$

- Then, expand $H_{\text{int.}}^{\text{QED}} = e\boldsymbol{p}_{\mathrm{D}} \cdot \boldsymbol{A}(\boldsymbol{x}, 0)$ in terms of Pauli matrices,

$$H_{\text{int.}}^{\text{QED}} = \alpha\mathbf{I} + \beta\sigma_z + \gamma\sigma_x + \delta\sigma_y.$$

  Here $\alpha, \beta, \gamma, \delta$ are **field operators.**
- Rearrange and forget about α, β terms to get back UdW Hamiltonian with **complex** profile

$$\boldsymbol{F}(\boldsymbol{x}) = -i\Psi_e^*(\boldsymbol{x})\nabla\Psi_g(\boldsymbol{x})$$

From QED to UdW

- To put it in a UdW-like form, restrict atomic dynamics to a two-level system,

$$\mathcal{H} \approx \{|\Psi_g\rangle, |\Psi_e\rangle\}, \quad \boldsymbol{p}_{\mathrm{D}} \approx \begin{pmatrix} \langle\Psi_g|\boldsymbol{p}_{\mathrm{D}}|\Psi_g\rangle & \langle\Psi_g|\boldsymbol{p}_{\mathrm{D}}|\Psi_e\rangle \\ \langle\Psi_e|\boldsymbol{p}_{\mathrm{D}}|\Psi_g\rangle & \langle\Psi_e|\boldsymbol{p}_{\mathrm{D}}|\Psi_e\rangle \end{pmatrix}.$$

- Then, expand $H_{\text{int.}}^{\text{QED}} = e\boldsymbol{p}_{\mathrm{D}} \cdot \boldsymbol{A}(\boldsymbol{x}, 0)$ in terms of Pauli matrices,

$$H_{\text{int.}}^{\text{QED}} = \alpha\mathbf{I} + \beta\sigma_z + \gamma\sigma_x + \delta\sigma_y.$$

  Here $\alpha, \beta, \gamma, \delta$ are **field operators.**
- Rearrange and forget about α, β terms to get back UdW Hamiltonian with **complex** profile

$$\boldsymbol{F}(\boldsymbol{x}) = -i\Psi_e^*(\boldsymbol{x})\boldsymbol{\nabla}\Psi_g(\boldsymbol{x})$$

What about α and β?

- α can be reabsorbed in the free field Hamiltonian exactly by defining a new set of field modes

$$b_{\boldsymbol{p}}^{\lambda} = a_{\boldsymbol{p}}^{\lambda} + eF[\lambda, \Psi_e(\boldsymbol{x}), \Psi_g(\boldsymbol{x})]$$

  This induces some constant corrections to β, γ, δ which can be dealt with.

- We cannot kill β in the same way. However, it vanishes if $\Psi_g(\boldsymbol{x}) = \Psi_e(\boldsymbol{x})$.
  - Use a strong **magnetic field** to get energy splitting with $\Psi_g(\boldsymbol{x}) = \Psi_e(\boldsymbol{x})$
  - The gap is $\hbar\Omega = \mu_{\mathrm{B}} B$
  - Perturbation theory regime: $E < \mu_{\mathrm{B}} B/(ed)$, where $d \equiv$ characteristic atomic length scale.

What about α and β?

- α can be reabsorbed in the free field Hamiltonian exactly by defining a new set of field modes

$$b_{\boldsymbol{p}}^{\lambda} = a_{\boldsymbol{p}}^{\lambda} + eF[\lambda, \Psi_e(\boldsymbol{x}), \Psi_g(\boldsymbol{x})]$$

  This induces some constant corrections to β, γ, δ which can be dealt with.
- We cannot kill β in the same way. However, it vanishes if $\Psi_g(\boldsymbol{x}) = \Psi_e(\boldsymbol{x})$.
  - Use a strong **magnetic field** to get energy splitting with $\Psi_g(\boldsymbol{x}) = \Psi_e(\boldsymbol{x})$
  - The gap is $\hbar\Omega = \mu_{\mathrm{B}}B$
  - Perturbation theory regime: $E < \mu_{\mathrm{B}}B/(ed)$, where $d \equiv$ characteristic atomic length scale.

Spectral response

- Detector is tuned to field frequency Ω.
- For an accelerated detector, Doppler shift $\Omega_{\text{obs.}} = \Omega e^{-\tau a/c}$.
- Spatial extension introduces a nontrivial spectral response (form factor),

$$\hat{F}(\boldsymbol{k}) = \int d\boldsymbol{x}\, F(\boldsymbol{x}) e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}$$

Detection prob. at momentum $k = \frac{\Omega}{c}$ is $\propto |F(\boldsymbol{k})|^2$.

Spectral response

- Detector is tuned to field frequency Ω.
- For an accelerated detector, Doppler shift $\Omega_{\text{obs.}} = \Omega e^{-\tau a/c}$.
- Spatial extension introduces a nontrivial spectral response (form factor),

$$\hat{F}(\boldsymbol{k}) = \int d\boldsymbol{x}\, F(\boldsymbol{x}) e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}$$

Detection prob. at momentum $k = \frac{\Omega}{c}$ is $\propto |F(\boldsymbol{k})|^2$.

Spectral response

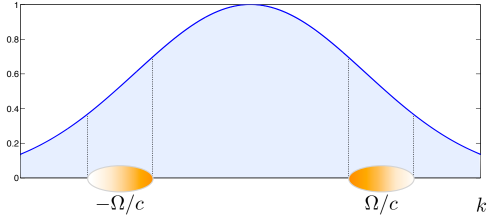
- Detector is tuned to field frequency Ω.
- For an accelerated detector, Doppler shift $\Omega_{\text{obs.}} = \Omega e^{-\tau a/c}$.
- Spatial extension introduces a nontrivial spectral response (form factor),

$$\hat{F}(\boldsymbol{k}) = \int d\boldsymbol{x}\ F(\boldsymbol{x}) e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}$$

Detection prob. at momentum $k = \frac{\Omega}{c}$ is $\propto |F(\boldsymbol{k})|^2$.

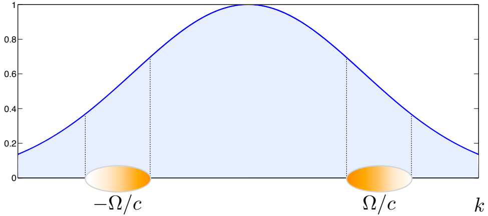
Problems ahead

- What we would like:

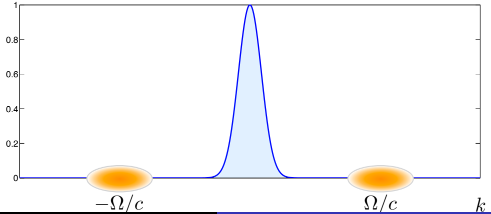


Problems ahead

- What we would like:



- What we get:



Solution: oscillations

- Induce oscillations in $\Psi(x)$ such that[2]

$$F_{\text{osc.}}(x) = F(x)\cos\left(\frac{\Omega x}{c}\right)$$

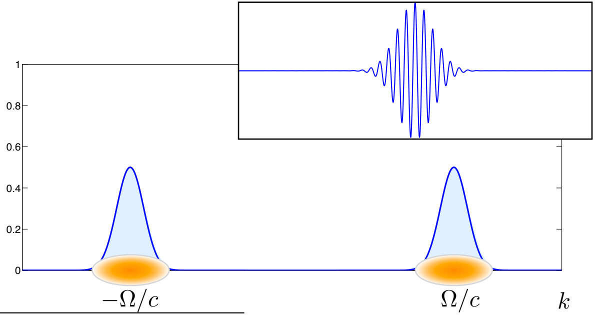
[2] See also A. Lee, I. Fuentes, quant-ph/1211.5261 (2012)

Solution: oscillations

- Induce oscillations in $\Psi(x)$ such that[2]

$$F_{\text{osc.}}(x) = F(x)\cos\left(\frac{\Omega x}{c}\right)$$

- Then



[2] See also A. Lee, I. Fuentes, quant-ph/1211.5261 (2012)

Summary

- We have derived the UdW interaction from first principles.
- To be realistic one needs **complex** smearing functions.
- Doppler shift in accelerated detectors can be dealt with through oscillations in the smearing function.

Thank you very much!