Possible and Impossible Interventions on Quantum Fields

Based on work done in collaboration with:
Dionigi Benincasa, Leron Borsten, Fay Dowker
arXiv:1206.5205

April 3, 2013

Show me the bits!

In a typical quantum information processing (QIP) scheme classical agents use a quantum system to encode, process and communicate information, (classical) bits.

In any such protocol we need to understand how the classical bits are encoded physically in the quantum system and how they are read out: show me the bits!

A related question is: How do agents perform unitary transformations and other sorts of operations on the qubits? How are the bits read out at the end by making measurements on some of the qubits? So a related demand is: show me the (physical) qubits!

This is particularly challenging when investigating protocols in a relativistic spacetime, taking into account the locations in spacetime of the actions of the external agents on the quantum system.

Main Points

In relativistic quantum field theory, there is no universally applicable rule for calculating the probability for the outcome of a sequence of interventions.

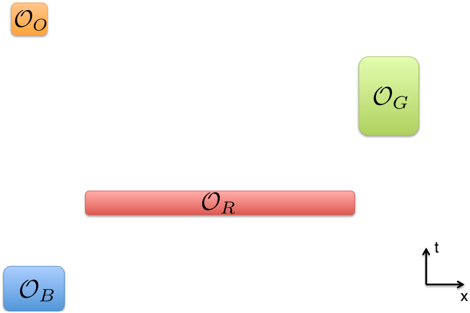
A straight-forward extension of the non-relativistic rule leads to superluminal signalling when certain ideal measurements and unitary transformations are performed.

To understand what is going on physically, it is important to model physical interventions on quantum fields carefully – e.g. detector models. We find that certain detector models lead to superluminal signalling, too.

We could take these issues as a guide in our foundational questions — they push us to seek a framework for closed relativistic quantum systems including detectors, in which experimental, measurement-like situations can be analysed fully quantum mechanically.

Sequences of Operations in Spacetime

Consider a collection of spacetime regions $\{\mathcal{O}_i\}$ and a sequence of operations in these regions.



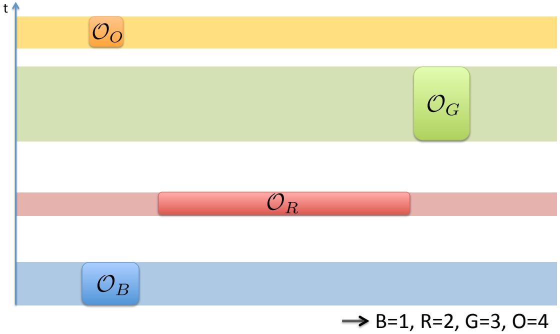
To make predictions about the outcome of a sequence of operations, we order the operations in time and use the standard procedure (evolve, collapse,. . .).

Sequences of ideal measurements in *non-relativistic* spacetime

Corresponding to each region $\mathcal{O}_i$ is an observable A_i and a particular possible outcome α_i associated with a projection operator P_i. The initial state is ρ.

We can then calculate the probability for a sequence of measurement outcomes:

$$\mathrm{Tr}(P_n \ldots P_1 \rho P_1 \ldots P_n).$$

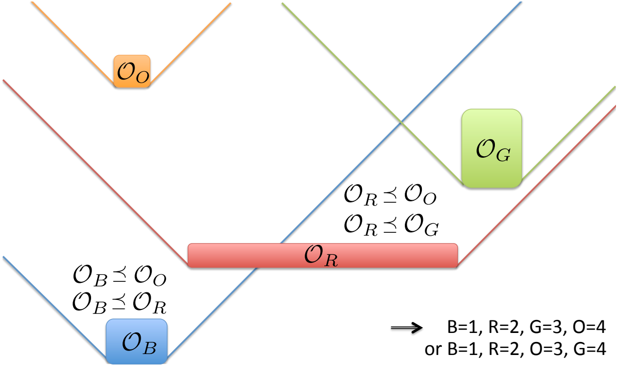


The order for the projectors P_i is obtained from the order of the regions:

- Let $\mathcal{O}_i \preceq \mathcal{O}_j$ iff some point in O_i lies to the past of some point $\mathcal{O}_j$.
- This is a *linear* order, so we can use it to order the P_i.

Sequences of ideal measurements in *relativistic* spacetime

What is the analog formula in a relativistic spacetime? Use the relativistic causal structure.

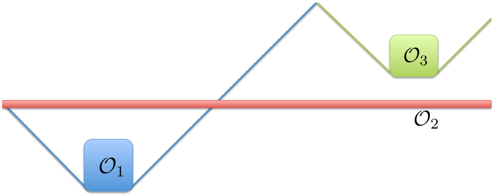


Sorkin (1993): use the same formula $\mathrm{Tr}(P_n \dots P_1 \rho P_1 \dots P_n)$, and use the rule

- $\mathcal{O}_i \preceq \mathcal{O}_j$ iff some point in O_j is to the causal past of some point in $\mathcal{O}_j$.
- Take the transitive closure of $\preceq$. (Non-unique but OK.)
- If the transitive closure is a partial order, we can use it to label the regions.

Measurement of a one-particle state

Let $\phi(x)$ be a free massive scalar field in $3+1$-dimensional Minkowski spacetime. Denote a 1-particle state of momentum $\mathbf{k}$ by $|1\rangle = a^\dagger_{\mathbf{k}}|0\rangle$.
We work in the Heisenberg picture and the field is in the vacuum state $|0\rangle$.



Consider the following protocol (Sorkin 1993):

- In $\mathcal{O}_1$, perform a unitary kick $U = e^{i\lambda\phi(x)}$, $x \in \mathcal{O}_1$
- In $\mathcal{O}_2$, measure the observable $P = |1\rangle\langle 1|$
- In $\mathcal{O}_3$, measure the expectation value $\langle\phi(y)\rangle$, $y \in \mathcal{O}_3$.

Let's define the "strength" of the signal from x to y: $S(x,y) \equiv \left.\dfrac{d\langle\phi(y)\rangle}{d\lambda}\right|_{\lambda=0}$

Measurement of a one-particle state

A quick calculation shows: $S(x,y) = \frac{1}{\omega_k (2\pi)^3} \sin[k_\mu (x^\mu - y^\mu)]$.

This is non-zero for most spacelike x and y and it does not decay in time at all.

This is not that surprising: the measurement in $\mathcal{O}_2$ is highly non-local.

Perhaps more surprisingly, the superluminal signal persists when $|1\rangle$ is replaced by a "localised" wave packet state (Benincasa, Borsten, MB, Dowker 2012).

Let's set $m = 0$ and use a Gaussian one-particle wave-packet state peaked at the origin at $t = 0$:

$$|1\rangle = (\pi\sigma^2)^{-\frac{3}{4}} \int d^d k \, e^{-\frac{(\mathbf{k}-\mathbf{k}_0)^2}{2\sigma^2}} a^\dagger_{\mathbf{k}} |0\rangle,$$

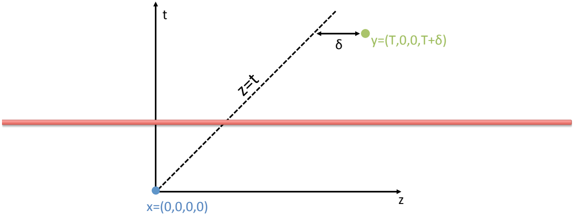
where $\mathbf{k}_0$ is its mean momentum, σ is the spread in momentum and $|\mathbf{k}_0| \gg \sigma$.

It can be shown that $S(x,y) \propto \text{Im}\,[\psi(y)]$ where $\psi(y) = \langle 0|\phi(y)|1\rangle$ is the "one-particle wavefunction".

Measurement of a one-particle wave packet state

In $1+1$ dimensions: if the packet has support on momenta in the positive spatial direction, then $\psi(y) = \psi(y+\xi)$ for any null $\xi^\mu \propto (k_0, k_0)$: no spread and the superluminal signal persists undiminished for arbitrary times.

In $3+1$ dimensions, the wave packet spreads due to dispersion and the calculation of $S(x,y)$ is not so simple.



Let's restrict ourselves to the $z-t$ plane: take $\mathbf{k}_0 = (0,0,k_0)$, $x = (0,0,0,0)$ and $y = (T,0,0,T+\delta)$. For $0 < \delta < 1/\sigma$ we find at late times $T \gg k_0/\sigma^2$:

$$S(x,y) \sim \sqrt{k_0/\sigma}\cos(k_0\delta)t^{-1}.$$

The signal decays but is non-zero for spacelike separated x and y.

Different rules

The same no-go result applies to a non-local unitary transformation done in $\mathcal{O}_2$.

Modelling external interventions on quantum fields as ideal measurements and unitary transformations in a straightfoward generalisation of the rules for non-relativistic quantum mechanics fails.

One response is to restrict the domain of validity of the generalisation:

- Restrict the regions $\{\mathcal{O}_i\}$
- Restrict the observables

A more physical approach: attempt to model interactions by constructing detector models.

Physical Detector Models

The construction of realistic detector models are one important step in understanding what goes on in interactions between quantum systems and an external agents.

Unruh-DeWitt detectors, e.g.:

$$H_{int} \propto \lambda(\tau)\phi(\tau, \mathbf{x}(\tau))(de^{-i\sigma\tau} + d^{\dagger}e^{i\sigma\tau})$$

Detectors with finite mode couplings:

$$H_{int} \propto \lambda(\tau)\left[\sum_{i=1}^{N}(a_{\mathbf{k}_i}e^{-i\omega_i t+\mathbf{x}(\tau)} + a_{\mathbf{k}_i}^{\dagger}e^{i\omega_i t-\mathbf{x}(\tau)})\right](de^{-i\sigma\tau} + d^{\dagger}e^{i\sigma\tau})$$

These models allow us to investigate the interaction of external agents with quantum fields more carefully. How do the findings above apply to detector models?

A simple protocol involving two detectors provides a test of whether superluminal information transfer results from the probabiliy formula.

Superluminal signals from detectors

Picture two inertial detectors at rest at x_1 and x_2 with couplings $\lambda_1(t)$ and $\lambda_2(t)$.The total interaction Hamiltonian is

$$H_{int}(t) = H_{1,int}(t) + H_{2,int}(t).$$

Both detectors and the scalar field are initially in their free ground states at $t = 0$ and the detectors are switched off before $t = 0$: $\lambda_i(t) = 0$ for $t < 0$.

At time $T > 0$ we measure for detector 1 the expectation value of the energy.

For a particular case of the finite-mode coupling detector, the evolution can be solved exactly (Dragan and Fuentes 2010). It can then be shown that it depends on λ_2 even when $T < |x_2 - x_1|$ for the finite mode detector.

The reason: $[H_{1,int}(t_1), H_{2,int}(t_2)] \neq 0$ for almost every pair of spacetime points along the trajectories of two detectors which are spacelike separated, so the unitary evolution operator does not separate for the two detectors.

The Unruh-deWitt detector is safe from this because it couples to the field itself — whether UD detectors are good models of realistic detectors that can be used in quantum information processing is an open question.

Conclusions

A simple extension of the non-relativistic rule for calculating the outcome of a sequence of operations has given rise to a conflict between locality and the concepts of ideal measurements and unitary interventions.

What are accurate models of physical interventions on relativistic quantum fields in quasi-local regions of Minkowski spacetime and what applications might they have in quantum information processing? Localised UD-type detectors form one class of models: Are they realistic?

The struggle to describe measurements of relativistic quantum fields in a physical way also reveals the limitations of the canonical and operational approach to quantum theory.

As a framework for closed quantum systems which deals directly with spacetime events, the path integral approach is eminently suitable for the investigation of measurements on relativistic quantum fields in Minkowski spacetime.

Side Note: Cavities

In a cavity, ideal measurements of observables such as particle number *can* be done (Johnson et. al. 2010).

For example, there is a well-known, successful model of an atom-qubit interacting with QED in a cavity which is of the form investigated above: the *Jaynes-Cummings model*.

No conflict with the results above: the Jaynes-Cummings model is a *phenomenological* model which applies only on time scales many orders of magnitude larger than the light crossing time of the cavity.

The Jaynes-Cummings Hamiltonian and its relatives cannot model an atom-qubit coupled to a quantum field in Minkowski spacetime, or in any spacetime where two atom-qubits can be placed at distances larger than the timescale on which the detector model is valid.