

Quantum Frameness for Charge-Parity-Time Inversion Symmetry

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- 1 Superselection, Frameness, and Quantum Information
- 2 Charge, Parity, and Time (CPT) Inversion symmetry
- 3 CPT Frameness
- 4 Conclusion

Outline

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Superselection Rule (SSR)

Wick, Wightman & Wigner, *Phys. Rev.* **88**, 101 (1952).

The limitations to the concept of parity of Q-mechanical states and, in particular, of intrinsic parity of elementary particles are discussed. These limitations are shown to follow from “superselection rules,” i.e., from restrictions on the nature and scope of possible measurements. The existence of such superselection rules is proved for the case of spinor fields; it is also conjectured that a superselection rule operates between states of different total charge.

Superselection Rule (SSR)

Definition

Let $|i\rangle, |j\rangle \in \mathcal{H}$ be eigenstates of a conserved quantity C . A **superselection rule (SSR)** for C states that for all observables \mathcal{O}

$$\langle i | \mathcal{O} | j \rangle = 0 \quad (1)$$

- Set of observable quantities a **strict subset** of all Hermitian operators
- The state $|\psi\rangle = c_i |i\rangle + c_j |j\rangle$ is **operationally indistinguishable** from $\rho = |c_i|^2 |i\rangle\langle i| + |c_j|^2 |j\rangle\langle j|$
- No coherences between eigenspaces corresponding to different values of C

$$\mathcal{H} = \bigoplus_{\lambda} \mathcal{H}^{(\lambda)}$$

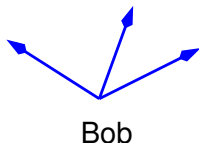
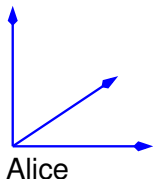
Reference Frames and SSRs

Quantum States (Nielsen and Chuang)

Associated to any physical systems is... The system is **completely** described by its state vector, $|\psi\rangle$, which is a unit vector in the system's state space.

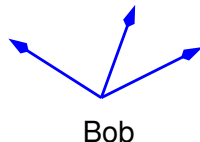
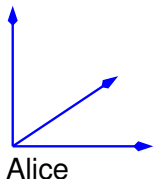
- Let the spin of an electron be $|0\rangle_x \equiv \frac{1}{\sqrt{2}}(|0\rangle_z + |1\rangle_z)$
- What does “up in the x -direction” mean?
- $|0\rangle_x$ contains information about the R.F. relative to which the electron's spin was prepared [[Int. J. Quant. Inf. 4, 17 \(2006\)](#)]

Reference Frames and SSRs



- Set of all possible transformations of a R.F. forms a **symmetry group**, G
- A and B's reference frames are related by some $g \in G$
- A and B have **complete ignorance** about $g \in G$

Reference Frames and SSRs



G -SSR

Any state $\rho = |\psi\rangle\langle\psi|$ in A's R.F. is **described** by B as

$$\mathcal{G}[\rho] \equiv \int U_g \rho U_g^\dagger dg = \bigoplus_q \rho^{(q)}, \quad (2)$$

where $U : G \rightarrow GL(\mathcal{H})$, dg is the Haar measure, and q are the **irreducible representations** (irreps) of G .

Resource Theory of Frameness

- Lack of a shared RF implies that Alice can
 - 1 prepare only ***G*-invariant** states

$$[\rho, U_g] = 0 \quad \forall g \in G$$

- 2 perform only ***G*-invariant** operations

$$\mathcal{E} [U_g(\rho)U_g^\dagger] = U_g (\mathcal{E}[\rho]) U_g^\dagger, \quad \forall g \in G$$

w.r.t Bob's RF

- Restrictions can be partially alleviated if Alice possesses a **bounded-size token** of Bob's RF [[PRL 93, 180503 \(2004\)](#)]

Resource Theory of Frameness

Frameness

- G -invariant operations
- Bounded-sized tokens of R.F.
- G -invariant states

Entanglement

- LOCC
- Entanglement
- Separable states

In terms of Tasks

- Frameness Purification
- Frameness Distillation
- Frameness interconversion

Motivation and Results

- Develop SSR for CPT invariance
- Construct frameness for spins 0 , $\frac{1}{2}$ and 1 (and higher)
- Quantify CPT frameness resources

Novelty

- Treating CPT as indecomposable, with strictly unitary representations
- Show that CPT-SSR has to be alleviated to execute Q information tasks with spin-0 particles
- Demonstrate that Q information tasks can be executed for higher spinors even in the presence of CPT-SSR

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The Problem with CPT

- CPT symmetry concerns the simultaneous inversion of
 - charge (C) with operator C
 - parity (P) with operator P (NJP 10, 033023 (2008))
 - time (T) with operator T (JMP 50, 102105 (2009))
- Fundamental symmetry - due to Lorentz covariance & linearity of Q mech.

Problem: C and T are anti-unitary

- Under CPT-invariant operations the set of CPT-invariant states must map onto itself
- Anti-unitary representations are basis dependent \implies non-resources evolve to resources
- No conserved quantity can be assigned to an anti-unitary representation

Our solution to CPT

Definition (Conserved internal symmetries)

Let A be the total baryon number, L the total lepton number, and Q the total charge.

$$p := Q + (A - L)$$

- Represent CPT as **indecomposable**

$$CPT |p, s\rangle = e^{i\theta} |-p, -s\rangle, \quad CPT^2 |p, s\rangle = e^{i2\theta} |p, s\rangle$$

- Anti-matter is the mirror image of matter (Feynman-Stueckelberg)
- Seek a projective **unitary representation** of $\{\mathbf{1}, CPT\}$ on the space of states $\mathcal{H} \equiv \text{span}\{|p, s\rangle\}$

Massive Spin-0 Bosons

Klein-Gordon Equation:

$$\left(\square - \frac{m^2 c^2}{\hbar^2} \right) \psi = 0$$

- Solutions are $\{|\pm p\rangle\}$ with $+$ identified with particles and $-$ with anti-particles
- $CPT = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
- Eigenvalues are ± 1
- Eigenstates (non-resources) are $|\pm\rangle \equiv \frac{1}{\sqrt{2}} (|p\rangle \pm |-p\rangle)$.

$\{\mathbf{1}, CPT\}$ on spinless massive particles is a 2-d unitary representation of \mathbb{Z}_2

Massive Spin-1/2 Particles

Dirac Equation:

$$(i\hbar \underbrace{\gamma^\mu \partial_\mu}_{\not{\partial}} + mc)\psi = 0, \quad \gamma^0 = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix}, \quad \boldsymbol{\gamma} = \begin{pmatrix} \mathbf{0} & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & \mathbf{0} \end{pmatrix}$$

Solutions to Dirac equation:

$$|p, 1/2\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad |p, -1/2\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$|-p, 1/2\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad |-p, -1/2\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

Massive Spin-1/2 Particles

- $CPT = \gamma^0 \gamma^1 = \begin{pmatrix} 0 & \sigma_x \\ \sigma_x & 0 \end{pmatrix}$
- Eigenvalues: ± 1 both with multiplicity 2
- Eigenvectors:
 - $|\pm, 0\rangle := \frac{1}{\sqrt{2}} (|p, \frac{1}{2}\rangle \pm |-p, -\frac{1}{2}\rangle)$
 - $|\pm, 1\rangle := \frac{1}{\sqrt{2}} (|p, -\frac{1}{2}\rangle \pm |-p, \frac{1}{2}\rangle)$
- Majorana spinors are also invariant under this action when given as solutions to the Majorana equation
 $i\hbar \not{\partial} \psi_c + mc\psi = 0$ where $\psi_c := i\psi^*$ in the Majorana basis.

$\{\mathbf{1}, CPT\}$ on massive spin-1/2 particles is a 4-d unitary representation of \mathbb{Z}_2

Massive Spin-1 Particles

WSG Equation:

$$[i\hbar\partial_\mu(\gamma^{\mu\nu} - g^{\mu\nu})i\hbar\partial_\nu + 2m_0^2c^2] \psi = 0$$

$\gamma^{\mu\nu}$ are the 6×6 matrices

$$\gamma^{ij=ji} = \begin{pmatrix} 0 & \delta_{ij}\mathbf{1} + M^{ij} + M^{ji} \\ \delta_{ij}\mathbf{1} + M^{ij} + M^{ji} & 0 \end{pmatrix}$$

$$\gamma^{0i} = \gamma^{i0} = \begin{pmatrix} 0 & S^i \\ -S^i & 0 \end{pmatrix}, \gamma^{00} = -\begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix}$$

The generalized spin matrices are $M^{ij} = iS^j iS^i$ and

$$S^1 = i \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, S^2 = i \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, S^3 = i \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Massive Spin-1 Particles

Solutions are:

$$|p, 1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, |p, 0\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, |p, -1\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

$$|-p, 1\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, |-p, 0\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, |-p, -1\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

Massive Spin-1 Particles

$$CPT = i^2 \gamma^{00} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

- Eigenvalues: ± 1 with multiplicity three
- Eigenvectors:

$$|\pm, 0\rangle = \frac{1}{\sqrt{2}}(|p, 1\rangle \pm |-p, -1\rangle),$$

$$|\pm, 1\rangle = \frac{1}{\sqrt{2}}(|p, 0\rangle \pm |-p, 0\rangle),$$

$$|\pm, 2\rangle = \frac{1}{\sqrt{2}}(|p, -1\rangle \pm |-p, 1\rangle).$$

$\{\mathbf{1}, CPT\}$ on massive spin-1 particles is a 6-d unitary representation of \mathbb{Z}_2

Massless Spin-1 Particles

Białynicki-Birula–Sipe equation:

$$i\hbar (\partial_0 + c\beta^3 S^j \partial_j) \psi = 0 \quad \beta^3 = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix}$$

- Solutions are six-component spinors: $\psi = \begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix}$ where Ψ_{\pm} represent opposite helicities
- Require auxiliary condition $\psi = \beta^1 \psi^*$ for $\beta^1 = \begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix}$
- Solutions same as for massive spin-1 WSG Equation
- $CPT = (\beta^3)^2 \beta^1$
- Transverse photon has nonzero spin: $|\pm p, 0\rangle$ unphysical.

CPT for higher-spin particles

- Employ Bargmann-Wigner equations
- These equations are built from Dirac equations
- Example:
 - Spin- $\frac{3}{2}$ particles obey Rarita-Schwinger equation
 - Solutions are 16-component spinors
 - Equivalent to four four-component spinors
 - Each spinor is a Dirac-equation solution

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Alignment of a CPT-RF

- A and B lack a shared RF for matter/anti-matter
- A and B **require** a shared RF
- However, any state ρ sent by A, B describes as

$$\left\{ \frac{1}{2}\rho, \frac{1}{2}CPT(\rho)CPT^\dagger \right\}$$

where $\{\mathbf{1}, CPT\}$ forms a **unitary representation** of \mathbb{Z}_2
whose irreps are given by ± 1

Question: What state should A sent and what measurement should B perform in order to be able to establish a shared RF for CPT?

Alignment of a CPT-RF

- A must prepare a state that is as **asymmetrical** (distinguishable) w.r.t CPT as possible
- X the random variable consisting of the elements of \mathbb{Z}_2
- Y the random variable associated with Bob's measurement outcome
- Bob holds the ensemble $\left\{ p_X, \rho(X) \equiv \left(U_X |\psi\rangle \langle \psi| U_X^\dagger \right)^{\otimes N} \right\}$
- The frameness of a quantum state is quantified by the **alignment rate**, $R(\psi)$ [NJP 14, 073022 (2012)]

CPT Frameness for Spin-0

- $CPT = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
- Eigenstates (non-resources) are $|\pm\rangle \equiv \frac{1}{\sqrt{2}} (|p\rangle \pm |-p\rangle)$
- Any $|\psi\rangle \in \mathcal{H}$ can be brought by **CPT-invariant** operations to the standard form

$$|\psi\rangle = \sqrt{q_+} |+\rangle + \sqrt{q_-} |-\rangle, \quad q_+ + q_- = 1$$

- $R(\psi) = -2 \log |q_+ - q_-|$
- $|\pm p\rangle$ are **ultimate resources** for CPT frameness
- π^\pm are resources. π^0 is a non-resource
- Alice and Bob can exchange **classical info** with spin-0 particles

CPT Frameness for Spin-1/2

$$|p, \frac{1}{2}\rangle_z = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, |p, \frac{-1}{2}\rangle_z = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, |-p, \frac{1}{2}\rangle_z = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, |-p, \frac{-1}{2}\rangle_z = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Positive Eigenstates

Negative Eigenstates

$$|+, 0\rangle \equiv \sqrt{\frac{1}{2}} (|p, \frac{1}{2}\rangle + |-p, \frac{-1}{2}\rangle)$$

$$|-, 0\rangle \equiv \sqrt{\frac{1}{2}} (|p, \frac{1}{2}\rangle - |-p, \frac{-1}{2}\rangle)$$

$$|+, 1\rangle \equiv \sqrt{\frac{1}{2}} (|p, \frac{-1}{2}\rangle + |-p, \frac{1}{2}\rangle)$$

$$|-, 1\rangle \equiv \sqrt{\frac{1}{2}} (|p, \frac{-1}{2}\rangle - |-p, \frac{1}{2}\rangle)$$

- Any $|\psi\rangle \in \mathcal{H}$ can be brought by **CPT-invariant** operations to the standard form

$$|\psi\rangle = \sqrt{q_+} |+, 0\rangle + \sqrt{q_-} |-, 0\rangle, \quad q_+ + q_- = 1$$

CPT Frameness for Spin-1/2

$$|p, \frac{1}{2}\rangle_z = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, |p, \frac{-1}{2}\rangle_z = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, |-p, \frac{1}{2}\rangle_z = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, |-p, \frac{-1}{2}\rangle_z = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Positive Eigenstates

$$|+, 0\rangle \equiv \sqrt{\frac{1}{2}} (|p, \frac{1}{2}\rangle + |-p, \frac{-1}{2}\rangle)$$

$$|+, 1\rangle \equiv \sqrt{\frac{1}{2}} (|p, \frac{-1}{2}\rangle + |-p, \frac{1}{2}\rangle)$$

Negative Eigenstates

$$|-, 0\rangle \equiv \sqrt{\frac{1}{2}} (|p, \frac{1}{2}\rangle - |-p, \frac{-1}{2}\rangle)$$

$$|-, 1\rangle \equiv \sqrt{\frac{1}{2}} (|p, \frac{-1}{2}\rangle - |-p, \frac{1}{2}\rangle)$$

- $R(\psi) = -2 \log |q_+ - q_-|$
- $|\vec{n}, \pm p\rangle$ are **ultimate** resources (electrons, positrons)

CPT Frameness for Spin-1/2

$$|p, \frac{1}{2}\rangle_z = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, |p, \frac{-1}{2}\rangle_z = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, |-p, \frac{1}{2}\rangle_z = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, |-p, \frac{-1}{2}\rangle_z = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Positive Eigenstates

Negative Eigenstates

$$|+, 0\rangle \equiv \sqrt{\frac{1}{2}} (|p, \frac{1}{2}\rangle + |-p, \frac{-1}{2}\rangle)$$

$$|-, 0\rangle \equiv \sqrt{\frac{1}{2}} (|p, \frac{1}{2}\rangle - |-p, \frac{-1}{2}\rangle)$$

$$|+, 1\rangle \equiv \sqrt{\frac{1}{2}} (|p, \frac{-1}{2}\rangle + |-p, \frac{1}{2}\rangle)$$

$$|-, 1\rangle \equiv \sqrt{\frac{1}{2}} (|p, \frac{-1}{2}\rangle - |-p, \frac{1}{2}\rangle)$$

- Alice and Bob can communicate **quantum information** even in the presence of CPT-SSR
- $|\psi\rangle = c_0 |+, 0\rangle + c_1 |+, 1\rangle, c_0, c_1 \in \mathbb{C}$

CPT Frameness for Higher Spins

- Under the action of $\{\mathbf{1}, CPT\}$ the state space of solutions of the relevant relativistic equations decomposes into two sectors $\mathcal{H} \equiv \mathcal{H}^{(+)} \oplus \mathcal{H}^{(-)}$
- Superpositions of eigenstates of CPT from different sectors are **resources**
- Resource states can always be brought via CPT-invariant operations to the **standard form**

$$|\psi\rangle = \sqrt{q_+} |+, 0\rangle + \sqrt{q_-} |-, 0\rangle$$
- For massless spin-1 particles resource states are superpositions of states with opposite helicities
- The amount of CPT frameness of a given state is given by $R(q) = -2 \log |q_+ - q_-| \rightarrow \infty$ as $q_{\pm} \rightarrow \frac{1}{2}$ (perfect resources)

Q.I. in the presence of CPT-SSR

Prepare & observe a spin-0 particle

- A prepares spin-0 $|c\rangle = c_+ |+\rangle + c_- |-\rangle$.
- B sees this state as completely decohered:

$$\rho = \frac{1}{2} \left(|\psi\rangle\langle\psi| + CPT|\psi\rangle\langle\psi|CPT^\dagger \right) = \sum_{\varepsilon \in \pm} |c_\varepsilon|^2 |\varepsilon\rangle\langle\varepsilon|.$$

- A & B must lift superselection to communicate Q info.

Prepare & observe a spin-1/2 particle

- A prepares spin-1/2 $|c\rangle = c_0 |+, 0\rangle + c_1 |+, 1\rangle$.
- Obviously CPT-invariant: B sees precisely Alice's state.
- Coherence between $|+, 0\rangle$ and $|+, 1\rangle$ preserved under CPT superselection.

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Conclusions

- First study of a SSR applicable in relativistic setting with charged particles
- Solved for spins 0, $\frac{1}{2}$, and 1 + method for higher spins
- CPT-SSR associated with the most fundamental symmetry
- CPT frameness enables CPT-SSR to be circumvented
- Q communication can be performed even with CPT-SSR by using Dirac fermions.

Interesting Questions

- Are there any SSRs for which no frameness exists?
- How do non-relativistic SSRs change in a relativistic setting (i.e. aligning a phase or Cartesian RF between two parties in relative motion)
- P -SSR where P is the Poincare group?