Quantum interferometric visibility as a witness for general relativistic proper time

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Introduction & Motivation: Quantum physics and general relativity have been individually confirmed to a very high precision. Combining both theories still remains a challenge. Growing precision of atom interferometry motivates experimental proposals, where the aim would be to detect general relativistic corrections to the Newtonian gravitational phase shift. However, such tests could only verify the non-Newtonian corrections in the Hamiltonian and not probe space-time geometry, the conceptual pillar of general relativity.

Results: Here we predict a quantum effect in interference experiments, which cannot be understood without invoking the general relativistic notion of proper time. We consider interference of a "clock" – a particle with an evolving internal degree of freedom - that will not only display a phase shift, but also reduce the visibility of the interference pattern to the extent to which the path information becomes available from reading out the proper time of the "clock". The observation of such a reduction in the visibility would be the first confirmation of a genuinely general relativistic effect in quantum mechanics.

Our result stresses that in quantum mechanics it makes no sense to speak about quantities without specifying how they are measured: considering proper time as a physical quantity on its own would imply that even without the "clock" interference is always lost, as which-path information is stored "somewhere".

Interpretation ambiguity of gravitationally induced phase shifts

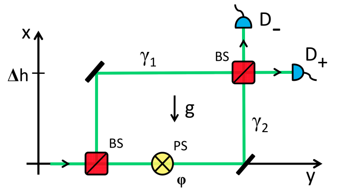

Fig. 1: Schematics of a Mach-Zehnder interferometer considered here. The setup inculdes two beam splitters, a phase shifter (PS) and two detectors $D_\pm$. PS gives a controllable phase difference φ between the two trajectories, labelled $\gamma_{1,2}$, which both lie in the x-y plane. A homogeneous gravitational field (g) is oriented along the x direction. The separation between the paths in the direction of the field is Δh.

Quantum state inside the interferometer of Fig.1

modes associated with the corresponding paths $\gamma_{1,2}$

$$|\Psi_{MZ}\rangle = \frac{1}{\sqrt{2}}\left(ie^{-i\phi_1}|r_1\rangle + e^{-i\phi_2+i\varphi}|r_2\rangle\right)$$

Probabilities of detection

$$P_\pm = \tfrac{1}{2} \pm \tfrac{1}{2}\cos\left(\Delta\phi + \varphi\right) \qquad \Delta\phi := \phi_1 - \phi_2$$

general relativity	non-relativistic quantum mechanics
gravity is a metric theory, proper time τ flows at different rates in different regions - depending on the space-time geometry	gravity is a potential (possibly non-Newtonian) force, there exists a global time paramter t, space-time is flat
$\phi_{i=1,2} \propto -mc^2 \int_{\gamma_i} \boxed{d\tau}$	$\phi_{i=1,2} \propto \int_{\gamma_i} dt\, \boxed{V_{eff}(x)}$
$\Delta\Phi$ originates in a general relativisitc time dilation	$\Delta\Phi$ comes from an Aharonov-Bohm effect (gravitational "version")

Interference of "clocks": idea

Use a "clock" - a system with an evolving in time degree of freedom (d.o.f.) – in the setup of Fig.1:

$$|\Psi_{MZ}\rangle = \frac{1}{\sqrt{2}}\left(i|r_1\rangle|\tau_1\rangle e^{-i\phi_1} + |r_2\rangle|\tau_2\rangle e^{-i\phi_2+i\varphi}\right)$$

state of the "clock", which followed the path $\gamma_{1,2}$

Probabilities of detection:

$$P_\pm = \tfrac{1}{2} \pm \tfrac{1}{2}\boxed{|\langle\tau_1|\tau_2\rangle|}\cos\left(\Delta\phi + \alpha + \varphi\right)$$

where: $\langle\tau_1|\tau_2\rangle = |\langle\tau_1|\tau_2\rangle|e^{i\alpha}$

Visibility of the interference pattern: $\mathcal{V} = |\langle\tau_1|\tau_2\rangle|$

Distunguishability of the paths: $\mathcal{D} = \sqrt{1 - |\langle\tau_1|\tau_2\rangle|^2}$

When proper time is treated operationally visibility is reduced by an amount given by the which-way information available form the "clock".

Proper time: new degree of freedom?

experimental visibility	possible explanation	current experimental status
$\mathcal{V}_m = 0$	proper time: quantum d.o.f., sharply defined	disproved in e.g. Ref. [1,2]
$0 < \mathcal{V}_m < \mathcal{V}_{QM}$	proper time: quantum d.o.f with uncertainty σ_τ	consistent with current data for $\sigma_\tau > \frac{\lvert\Delta\tau\rvert}{\sqrt{-8\ln(1-\Delta\mathcal{V})}}$
$\mathcal{V}_m = \mathcal{V}_{QM}$	proper time: not a quantum d.o.f. or has a very broad uncertainty	consistent with current data
$\mathcal{V}_m > \mathcal{V}_{QM}$	quantum interferometric complementarity does not hold when general relativistic effects become relevant	not tested

Table 1. Discussion of possible outcomes of the proposed interferometric experiment. Depending on the measured visibility V_m as compared to the value V_{QM} predicted by quantum mechanics different conclusions can be drawn regarding the possibility that proper time is a new quantum degree of freedom (d.o.f.). Assuming that the proper time d.o.f. is a Gaussian of width σ_τ, current interferometric experiments give bounds on possible σ_τ in terms of the proper time difference $\Delta\tau$ between the paths and the experimental error Δv of the visibility measurement.

Acknowledgements and References

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Results

Realization of the main idea with the "clock" d.o.f. implemented in internal states of a massive particle (neglecting finite size effects).

Hamiltonian

rest frame: evolution w.r.t. proper time τ

$$i\hbar\frac{\partial}{\partial\tau} \equiv H_{\circ}$$

laboratory frame: evolution w.r.t. coordinate time t

$$i\hbar\frac{\partial}{\partial t} = \dot\tau H_{\circ}$$

$$\dot\tau \equiv \frac{d\tau}{dt} = \sqrt{-g_{\mu\nu}\dot x^\mu \dot x^\nu}$$

metric

$$E = mc^2\frac{-g_{00}}{\sqrt{-g_{\mu\nu}\dot x^\mu \dot x^\nu}}$$

energy of a mass m in a space-time with metric $g_{\mu\nu}$ (canonical quantization of E gives Hamiltonian of the external d.o.f.)

Up to 2nd order in internal, kinetic and potential energy:

Earrth's gravitational potential

$$H_{\mathrm{Lab}} \simeq mc^2 + H_{\circ} + E_k^{GR} + \frac{\phi(x)}{c^2}\left(mc^2 + H_{\circ} + E^{GR}_{corr}\right)$$

$$E^{GR}_{corr} = \tfrac{1}{2}m\phi(x) - 3\frac{p^2}{2m},\quad E^{GR}_k = \frac{p^2}{2m}\left(1 + 3\left(\frac{p}{2mc}\right)^2 - \frac{1}{mc^2}H_{\circ}\right)$$

State in the path γ_i:

$$|\Psi_i\rangle = e^{-\frac{i}{\hbar}\int_{\gamma_i} dt\, \frac{\phi(x)}{c^2}\left(mc^2 + H_{\circ} + E^{GR}_{corr}\right)}|x^{in}\rangle|\tau^{in}\rangle$$

Initially particle is in a product sate of the path (x^{in}) and the internal (τ^{in}) d.o.f.

Probabilities of detection:

$$H_{\circ} = E_0|0\rangle\langle 0| + E_1|1\rangle\langle 1|$$

$$|\tau^{in}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

ΔE:=E1=E0, ΔV:=gΔh, ΔT: time for which the particle travels in superposition at constant heights, the term E^{GR}_{corr} is averaged over the two paths, the expectation value is taken with respect to the initial state of the "clock" d.o.f. (τ^{in})

$$P_\pm(\varphi, m, \Delta E, \Delta V, \Delta T) =$$

$$\frac{1}{2} \pm \frac{1}{2}\cos\left(\frac{\Delta E \Delta V \Delta T}{2\hbar c^2}\right)\cos\left(\left(mc^2 + \langle H_{\circ}\rangle + \bar E^{GR}_{corr}\right)\frac{\Delta V \Delta T}{\hbar c^2} + \varphi\right)$$

relative phase originating from the Newtonian potential

general relativistic corrections to the phase shift coming from the path d.o.f.

new effects appearing with the "clock" :	change in the visibility of the interference pattern $\mathcal{V} = \left\lvert\cos\left(\frac{\Delta E \Delta V \Delta T}{2\hbar c^2}\right)\right\rvert$	additional phase shift proportional to the average internal energy

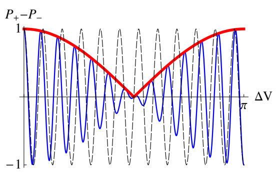

Fig. 2. Difference between the detection probabilities $P_\pm$ as a function of the potential difference ΔV between the paths (arbitrary units). The dashed, black line shows the situation without the "clock". In the case with the "clock" (blue line) we expect two new effects: an additional phase shift (corresponding to average internal energy of the "clock") and the modulation in the visibility (thick, red line) of the interference pattern according to the result above.

system	"clock"	ω [Hz]	$\Delta h \Delta T$ [ms] achieved	$\Delta h \Delta T$ [ms] requqired
atoms	hyperfine states	10^{15}	10^{-5}	10
electrons	spin precession	10^{13}	10^{-6}	10^{3}
molecules	vibrational modes	10^{12}	10^{-8}	10^{4}
neutrons	spin precession	10^{10}	10^{-6}	10^{6}

Table 2. Comparison of different possible systems for the observation of the reduced interferometric visibility, together with theoretically required and experimentally achieved parameters. For a "clock" with frequency $\omega = \Delta E/\hbar$, the required value of the parameter $\Delta h \Delta T$ (Δh being the separation between the interferometers arms and ΔT the time for which the particle travels in superposition at constant heights) for the full loss of the fringe visibility is given in the rightmost column. A constant gravitational acceleration g = 10 m/s² is assumed.

Conclusion:

- reduction of the interferometric visibility due to the accessible proper time difference provides a new paradigm for tests of genuine general relativistic effects in quantum mechanics,
- our approach allows to clarify the notion of proper time when taken in the quantum context - it stresses once again that only operationally well defined physical quantities have meaning in quantum mechanics,
- idea can lead to a conclusive test of theories in which proper time is assumed to be a quantum degree of freedom,
- result can give new insight into understanding the quantum-to-classical transition: gravitational time dilation provides a new mechanism of decoherence