Does cosmological particle creation alter the emission process by a local source?

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What people mean they say “you can’t get something from nothing”

$$a\,|0\rangle = 0 \implies \widehat{N}\,|0\rangle = a^{\dagger}a\,|0\rangle = 0 \implies \text{Particle eigennumber} = 0$$

Particle creation mechanism 1

$$H = -\frac{\hbar^2}{2m}\frac{\mathrm{d}^2}{\mathrm{d}x^2}+\frac{1}{2}m\omega^2 x^2,\ a = \frac{m\omega x + ip}{\sqrt{2m\hbar\omega}} \implies x = \frac{a + a^\dagger}{\sqrt{2m\omega/\hbar}}.$$

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$$a_{\mathrm{old}} = \frac{m\omega_{\mathrm{old}}x + ip}{\sqrt{2m\hbar\omega_{\mathrm{old}}}},\ a_{\mathrm{new}} = \frac{m\omega_{\mathrm{new}}x + ip}{\sqrt{2m\hbar\omega_{\mathrm{new}}}}$$

$$\implies a_{\mathrm{new}} = \alpha a_{\mathrm{old}} + \beta a^\dagger_{\mathrm{old}},$$

$$\alpha := \frac{\omega_{\mathrm{new}} + \omega_{\mathrm{old}}}{2\sqrt{\omega_{\mathrm{new}}\omega_{\mathrm{old}}}},\ \beta := \frac{\omega_{\mathrm{new}} - \omega_{\mathrm{old}}}{2\sqrt{\omega_{\mathrm{new}}\omega_{\mathrm{old}}}}.$$

CPC in one equation

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A lemma

$$f_{\mathrm{out}\,\mathbf{k}} = \frac{e^{-ik\cdot x}}{\sqrt{2\omega\,(2\pi)^3}}\ (+---)\ ,\ f_{\mathrm{in}\mathbf{k}} = \alpha_{\mathbf{k}} f_{\mathrm{out}\,\mathbf{k}} + \beta_{\mathbf{k}} f^*_{\mathrm{out}-\mathbf{k}}\,.$$

A sketch proof of the "one equation"
Stage 1: no source current

$$\phi(x) = \int \mathrm{d}^3\mathbf{k} \left[f_{\mathrm{in}\mathbf{k}}(x)\, a_{\mathrm{in}}(\mathbf{k}) + f^*_{\mathrm{in}\mathbf{k}}(x)\, a^\dagger_{\mathrm{in}}(\mathbf{k}) \right] = \int \mathrm{d}^3\mathbf{k} \left[f_{\mathrm{out}\mathbf{k}}(x)\, a_{\mathrm{out}}(\mathbf{k}) + f^*_{\mathrm{out}\mathbf{k}}(x)\, a^\dagger_{\mathrm{out}}(\mathbf{k}) \right],$$

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$$f_{\mathrm{in}\mathbf{k}} = \alpha_{\mathbf{k}} f_{\mathrm{out}\mathbf{k}} + \beta_{\mathbf{k}} f_{\mathrm{out}-\mathbf{k}}.$$

$$a_{\mathrm{out}}(\mathbf{k}) = \alpha_{\mathbf{k}} a_{\mathrm{in}}(\mathbf{k}) + \beta^*_{-\mathbf{k}} a^\dagger_{\mathrm{in}}(-\mathbf{k}).$$

A sketch proof of the "one equation"
Stage 2: include a source current

$$\left(\Box - m^2\right)\Delta\phi = \widehat{j}\,(x) = \int \mathrm{d}^4k\,\frac{e^{-ik\cdot x}}{\sqrt{(2\pi)^3}}\,j\,(k) = \int \mathrm{d}^4k\,\sqrt{2\omega}\,f_{\mathrm{out}\mathbf{k}}\,(x)\,j\,(k)$$

$$\Longrightarrow\ \Delta\phi = i\int \mathrm{d}^3\mathbf{k}\left[f_{\mathrm{out}\mathbf{k}}\,(x)\,j\,(k) - f^{*}_{\mathrm{out}\mathbf{k}}\,(x)\,j^{*}\,(k)\right].$$

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$$\phi = \int \mathrm{d}^3\mathbf{k}\,\left[f_{\mathrm{in}\mathbf{k}} a_{\mathrm{in}}\left(\mathbf{k}\right) + f^*_{\mathrm{in}\mathbf{k}} a^\dagger_{\mathrm{in}}\left(\mathbf{k}\right)\right] = \int \mathrm{d}^3\mathbf{k}\,\left[f_{\mathrm{out}\mathbf{k}}\left(a_{\mathrm{out}}\left(\mathbf{k}\right) + ij\left(k\right)\right) + f^*_{\mathrm{out}\mathbf{k}}\left(a^\dagger_{\mathrm{out}}\left(\mathbf{k}\right) - ij^*\left(k\right)\right)\right]$$

$$\Longrightarrow\ a_{\mathrm{out}}\left(\mathbf{k}\right) = \alpha_{\mathbf{k}} a_{\mathrm{in}}\left(\mathbf{k}\right) + \beta^*_{-\mathbf{k}} a^\dagger_{\mathrm{in}}\left(-\mathbf{k}\right) - ij\left(k\right).$$

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$$U = e^{-iF},\ F = \int \mathrm{d}^3\mathbf{k}\ \left(j a_q^\dagger + j^* a_q\right),\ a_q = \alpha a + \beta^* a^\dagger.$$

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$$[a_q,\,U] = -ijU \implies a_q U = U\,(a_q - ij) = U a_c.$$

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$$\sum_{n=0}^{N} \frac{(-1)^n}{n!\,(N-n)!} = N! \times \sum_{n=0}^{N} \binom{N}{n} (-1)^n = N! \times (1 + (-1))^N = N! \times 0^N = 0.$$

Compulsory "thank you" ending

What we knew before

- You can get something from nothing, all thanks to β
- This effect doesn't require a back-reacting source

What we know now

- Back-reaction doesn't induce quantum corrections to the probability distributions of boson numbers
- Fermions will take a bit more work