Quantum correlations with no causal order

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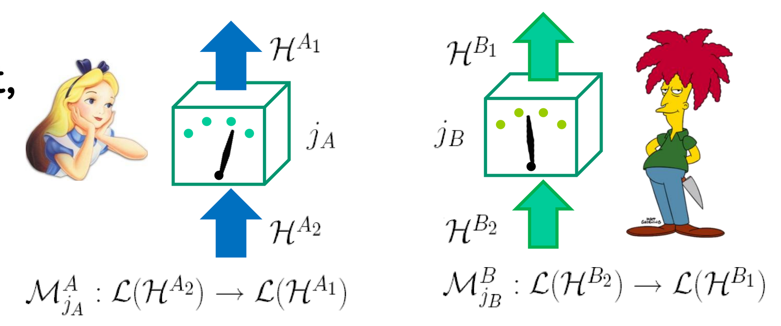
The correlations considered in quantum mechanics are typically assumed to arise from operations performed at specific space-time locations. This assumption puts restrictions on the possibility of signalling: no signal can be exchanged between space-like separated observers, while time-like separation allows signalling only in one direction, from the past to the future.
We pose the question whether the observed causal order between events is a necessary element of quantum theory. We first develop a formalism for multipartite quantum correlations that does not assume any underlying space-time or causal structure, but only that local agents are free to perform arbitrary quantum operations. All known situations, including non-signalling correlations between space-like separated observers, signalling ones between observers connected by a channel, as well as probabilistic mixtures of these, can be expressed in this formalism. We show that there exist situations allowed by the formalism where two experiments are neither causally ordered nor in a mixture of causal orders.

The most general bipartite quantum correlations

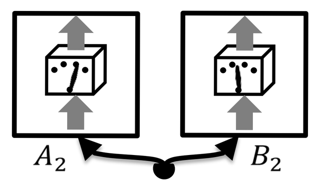
- **Each party performs a quantum measurement, most generally represented by a completely positive (CP) map.**



- **What is the most general probability distribution** $P(\mathcal{M}^A,\mathcal{M}^B)$ **they can observe?**

➢ **Shared quantum state**

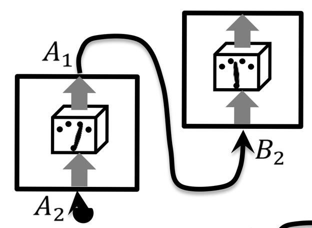
$$P(\mathcal{M}^A,\mathcal{M}^B)=\mathrm{Tr}\left[\mathcal{M}^A\otimes\mathcal{M}^B\left(\rho^{A_2B_2}\right)\right]$$



No signalling possible

➢ **Quantum channel**

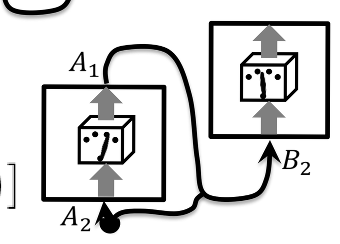
$$P(\mathcal{M}^A,\mathcal{M}^B)=\mathrm{Tr}\left[\mathcal{M}^B\circ\mathcal{E}\circ\mathcal{M}^A\left(\rho_0^{A_2}\right)\right]$$



Possibility of signalling from Alice to Bob

➢ **Quantum channel with memory**

$$P(\mathcal{M}^A,\mathcal{M}^B)=\mathrm{Tr}\left[(\mathcal{I}^A\otimes\mathcal{M}^B)\circ\mathcal{E}\circ(\mathcal{M}^A\otimes\mathcal{I}^B)\left(\rho^{A_2B_2}\right)\right]$$



Most general if Alice acts *before* Bob.

➢ **Also *probabilistic mixtures* are possible. For example, a channel that connects Alice to Bob with probability *q* and Bob to Alice with probability *(1-q)* gives probabilities**

$$P(\mathcal{M}^A,\mathcal{M}^B)=q\mathrm{Tr}\left[\mathcal{M}^B\circ\mathcal{E}\circ\mathcal{M}^A\left(\rho^{A_2}\right)\right]+(1-q)\mathrm{Tr}\left[\mathcal{M}^A\circ\mathcal{E}\circ\mathcal{M}^B\left(\rho^{B_2}\right)\right]$$

- **If no assumption is made about a global space-time in which the parties are immersed, can one imagine more general situations than those above?**

A natural assumption: probabilities are bilinear functions of the CP maps

True if the operations of each party are correctly described by quantum mechanics, in particular if probabilistic mixtures of maps are represented as convex combinations.

Formalism for general bipartite correlations

Choi-Jamiołkowski (CJ) isomorphism

CP maps $\mathcal{M}:\mathcal{L}(\mathcal{H}^2)\rightarrow\mathcal{L}(\mathcal{H}^1)$ ⟷ Bipartite positive operators $\rho^{12}_{\mathcal{M}}\in\mathcal{L}(\mathcal{H}^1)\otimes\mathcal{L}(\mathcal{H}^2)$

$$\rho^{12}_{\mathcal{M}}:=\mathcal{M}\otimes\mathcal{I}(|\Phi^+\rangle\langle\Phi^+|)\qquad|\Phi^+\rangle=\textstyle\sum_i|i\rangle|i\rangle\qquad|i\rangle\in\mathcal{H}^1$$

Representation of bilinear probabilities

$$P(\mathcal{M}^A,\mathcal{M}^B)=\mathrm{Tr}\left[W^{A_1A_2B_1B_2}\left(\rho^{A_1A_2}_{\mathcal{M}^A}\otimes\rho^{B_1B_2}_{\mathcal{M}^B}\right)\right]$$

$$W^{A_1A_2B_1B_2}\in\mathcal{L}(\mathcal{H}^{A_1}\otimes\mathcal{H}^{A_2}\otimes\mathcal{H}^{B_1}\otimes\mathcal{H}^{B_2})$$

Process matrix

Properties of process matrices

Positive: $W^{A_1A_2B_1B_2}\geq 0$ (assuming possibility of sharing entangled states)

Probability 1 for all CPTP maps: $\mathrm{Tr}\left[W^{A_1A_2B_1B_2}\left(\rho^{A_1A_2}_{\mathcal{E}^A}\rho^{B_1B_2}_{\mathcal{E}^B}\right)\right]=1$

$$\forall\rho^{A_1A_2},\rho^{B_1B_2}>0,\ \mathrm{Tr}_1\rho^{A_1A_2}=1\!\!1^{A_2},\ \mathrm{Tr}_1\rho^{B_1B_2}=1\!\!1^{B_2}$$

Characterizing property of the CJ operator of a CP, trace preserving (CPTP) map

All bipartite situations are represented by a process matrix!

Examples

➢ **Shared state** $W^{A_1A_2B_1B_2}=1\!\!1^{A_1B_1}\otimes\left(\rho^{A_2B_2}\right)^{\mathrm{T}}$

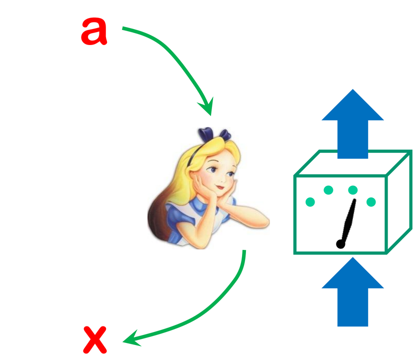
➢ **Channel A→ B** $W^{A_1A_2B_1B_2}=1\!\!1^{B_1}\otimes\left(\rho^{B_2A_1}\right)^{\mathrm{T}}\otimes\left(\rho_0^{A_2}\right)^{\mathrm{T}}$

➢ **Channel with memory** $W^{A_1A_2B_1B_2}=1\!\!1^{B_1}\otimes W^{A_1A_2B_2}\equiv W^{A\rightarrow B}$

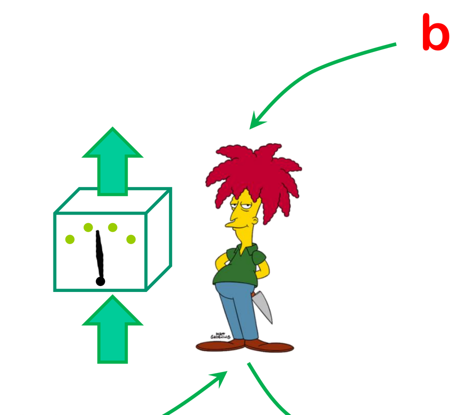
➢ ***Causally separable* process** $W^{A_1A_2B_1B_2}=qW^{A\rightarrow B}+(1-q)W^{B\rightarrow A}$

Are all process matrices causally separable?

A causal game



- **Each party Receives a bit and gives an estimate of the bit received by the other.**
- **Depending on the value of an additional bit b', Bob tries either to read a or to send b.**
- **They try to maximize the quantity**

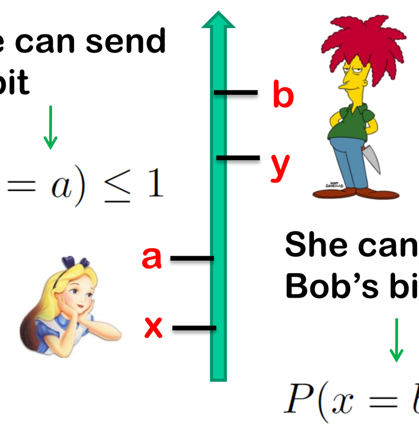


$$p_{succ}:=\frac{1}{2}\left[P(x=b|b'=0)+P(y=a|b'=1)\right]$$

With causal order

Alice can send her bit

$$P(y=a)\leq 1$$



She cannot read Bob's bit

$$P(x=b)\leq 1/2$$

Limited probability of success:

$$\tfrac{1}{2}\left[P(x=b)+P(y=a)\right]\leq\tfrac{3}{4}$$

Holds for every causally separable scenario.

Without causal order

A valid process matrix:

$$W^{A_1A_2B_1B_2}=\frac{1}{4}\left[1\!\!1+\frac{1}{\sqrt{2}}\left(\sigma_z^{A_1}\sigma_z^{B_2}+\sigma_z^{A_2}\sigma_z^{B_1}\sigma_x^{B_2}\right)\right]$$

Strategy:

Alice measures and encodes in the z basis ⇢ **Bob "sees"** $\widetilde{W}^{B_1B_2}=\frac{1}{2}\left(1\!\!1+(-1)^a\frac{1}{\sqrt{2}}\sigma_z^{B_2}\right)$

b'=1
Bob measures in the z basis ⇢ **He can read Alice's bit**

b'=0
Bob measures in the x basis, encodes in the z basis. ⇢ **Correlating the encoding with the outcome, he can "send"** $\widetilde{W}^{A_1A_2}=\frac{1}{2}\left(1\!\!1+(-1)^b\frac{1}{\sqrt{2}}\sigma_z^{A_2}\right)$

Alice can read Bob's bit

Probability of success:

$$\tfrac{1}{2}\left[P(y=a|b'=1)+P(x=b|b'=0)\right]=\tfrac{2+\sqrt{2}}{4}>\tfrac{3}{4}$$

The process matrix is not causally separable!

Conclusions

- We developed a formalism independent of any notion of background space-time or causal structure. This formalism allows a unified treatment of signalling and non-signalling .
- The formalism allows considering communication complexity tasks that do not presuppose space-like or time-like separated parties.
- We found a task for which causal order imposes a bound on the probability of success.
- We found correlations allowed by our formalism that violate the bound imposed by causal order. Such correlations cannot be understood as arising from causally ordered operations, nor as a mixture of causal orders.
- It is an open question whether these "non-causal" correlations can be observed in nature.
- The formalism we developed can be relevant for developing a theory that reconciles general relativity and quantum mechanics, since it can be expected that such theory should not have a fixed causal structure

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CoQuS ComplexQuantumSystems
FWF Der Wissenschaftsfonds.
Q|A|P

References:

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