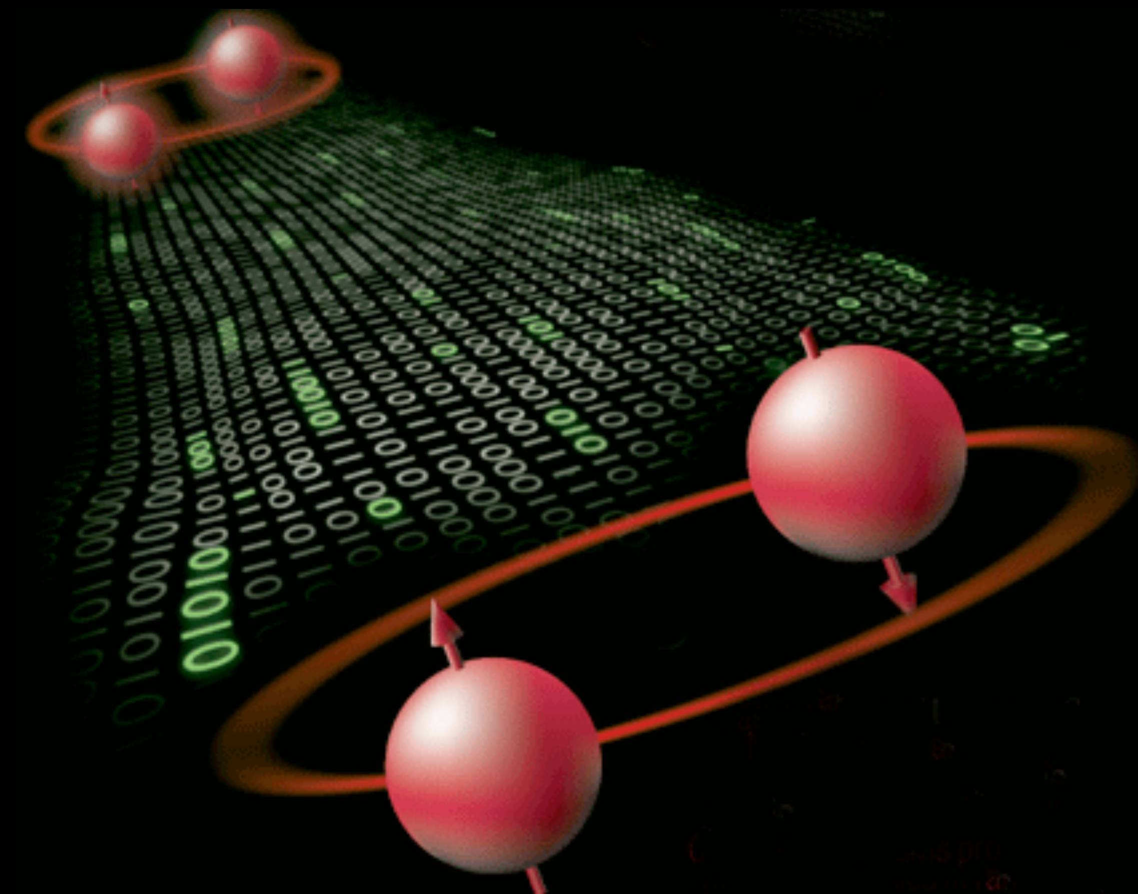
Entanglement entropy in integrable QFT



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Mainly based on:

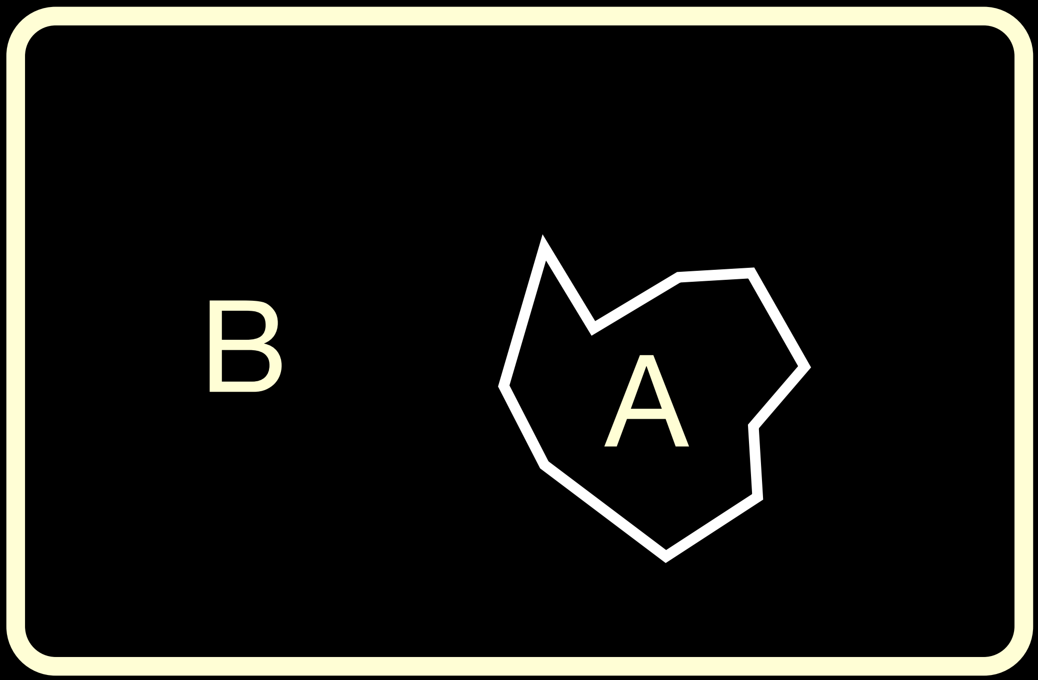
- J. Cardy, O. A. Castro Alvaredo, B. Doyon, J. Stat. Phys. 130, 129 (2008)
- O. A. Castro Alvaredo, B. Doyon, J. Phys. A 42 504006 (2009)
- B. Doyon, Phys. Rev. Lett. 102 031602 (2009)
- O. A. Castro Alvaredo, E. L., J. Phys. A 44 255401 (2011)
- O. A. Castro Alvaredo, B. Doyon, E. L., J. Phys. A 44 492003 (2011)
- E. L. , J. Phys. A 45 275401 (2012)
- A couple of works in progress...

Motivations:

We use QFT to describe the scaling limit of some many-body (lattice) models.

- Is there a universal measure of the amount of entanglement?
- Once found, what are its properties?

At T=0 the system is in its ground state $|G.S.\rangle$, with $\rho = |G.S.\rangle\langle G.S.|$.



$$\rho_A = \mathrm{Tr}_B \rho$$

$$S_A = -\mathrm{Tr}_A \rho_A \log \rho_A$$

Properties:

Even if it is clearly not the only suitable measure, entanglement entropy has the following very interesting properties:

- there exist many models for which it is possible to compute it analytically and numerically

- And most remarkably $S_A = S_B$ - - - - → Area Law.
(Srednicki, 1993)

For D=1 we have: $$S_A = \frac{c\nu}{6} \log \frac{l}{a}$$ (Holzhey, Larsen, Wilczek, 1994)

where ν is the number of “boundaries” between A and B.
We focus on the case $\nu = 2$

QFT as scaling limit:

The QFT representing the scaling limit of a non-critical quantum spin chain is a (1+1)-massive QFT

$$\langle S_i S_j \rangle \sim e^{-\frac{|i-j|}{\xi}} \qquad |i-j| \to \infty$$

We want to perform the limit

$$a \to 0$$

In order to have the same physics I have to consider

$$\xi \to \infty$$

at the same time.

QFT as scaling limit:

This limit must be supported by a "zoom-in" of all distances, in a way that I can identify

$$\lim_{g \to g_c} \xi^{2d} \langle S_i S_j \rangle = \langle 0 | O_s(x) O_s(x') | 0 \rangle \sim \phi(m|x - x'|)$$

This is a two-point function of a massive Euclidean relativistic QFT

identifying

$$ia \to x$$

$$\xi a \to m^{-1}$$

The method:

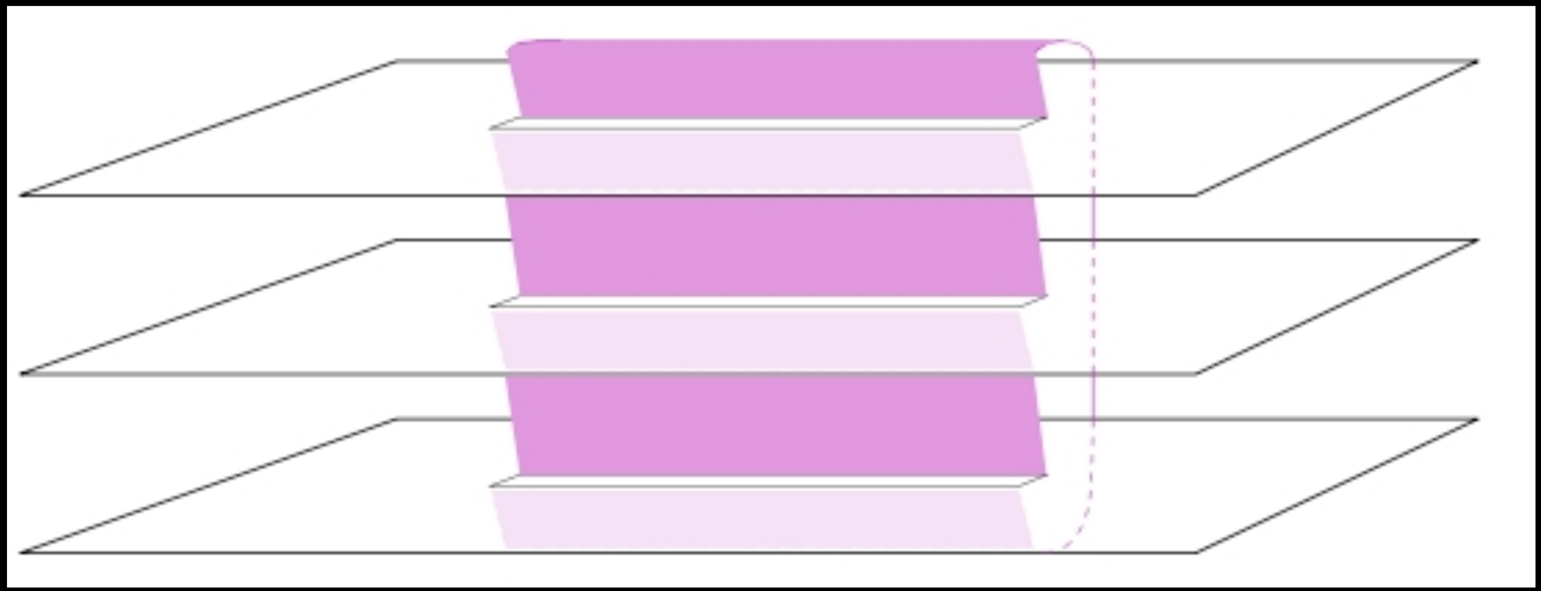
Focusing on an Euclidean QFT we use the replica trick:

$$S_A = -\lim_{n\to 1} \frac{\partial}{\partial n} \mathrm{Tr}_A \rho_A^n$$

(Callan, Wilczek 1994)

for n integer

$$\mathrm{Tr}_A \rho_A^n \sim \int [\mathcal{D}\phi]_{\mathcal{M}_n} e^{-\int_{\mathcal{M}_n} d^2x \mathcal{L}[\phi]} = Z_n$$

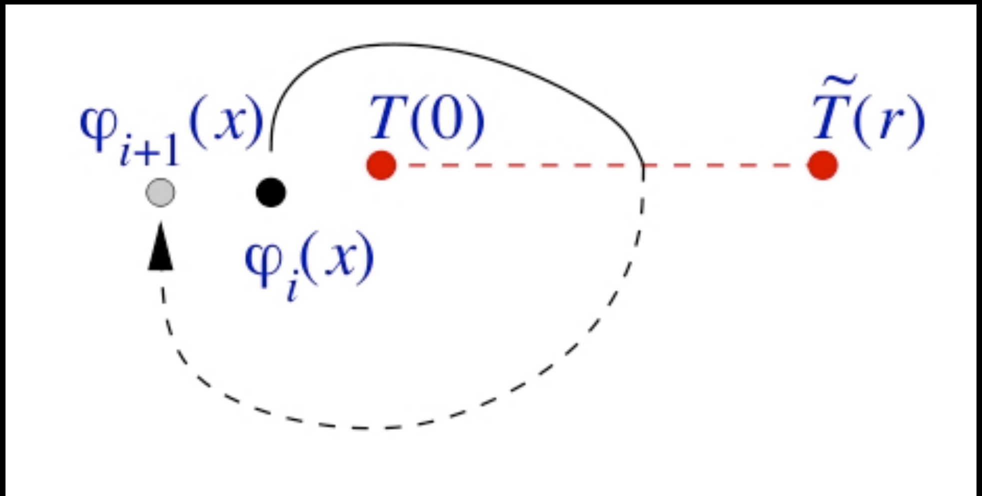


(Pictures thank to B. Doyon)

The method:

The method we use for implementing the manifold is by some twist fields. First we consider n disconnected copies

$$\mathcal{L}^n\left[\phi_1,\phi_2,...,\phi_n\right](x) = \mathcal{L}\left[\phi_1(x)\right] + \mathcal{L}\left[\phi_2(x)\right] + ... + \mathcal{L}\left[\phi_n(x)\right]$$



$$Z_n \sim \langle \mathcal{T}(0)\tilde{\mathcal{T}}(r)\rangle$$

(Picture thank to B. Doyon)

Main analytical results:

For large distance is particular convenient to use a form factor expansion. The main challenges are:

- the actual evaluation of form factors

$$F^{\mathcal{T}}_{\mu_1 \ldots \mu_k}(\theta_1, \ldots, \theta_k) = \langle 0|\mathcal{T}(0)|\theta_1, \ldots, \theta_k\rangle^{\text{in}}_{\mu_1, \ldots, \mu_k}$$

- The analytical continuation for non integer number of copies

Eventually up to two particle form factors the result is

$$S_A \sim -\frac{c}{3}\log(\epsilon m_1) + U - \frac{1}{8}\sum_{\alpha=1}^{\#\text{part}} K_0(2m_\alpha r) + O\left(e^{-3m_1 r}\right)$$

Numerical results:

The first model we use to check our results is the Ising spin chain

$$H = -\frac{J}{2}\sum_{i=1}^{N}\left(\sigma_i^z\sigma_{i+1}^z + h\sigma_i^x\right).$$

Q.C.P. h=1

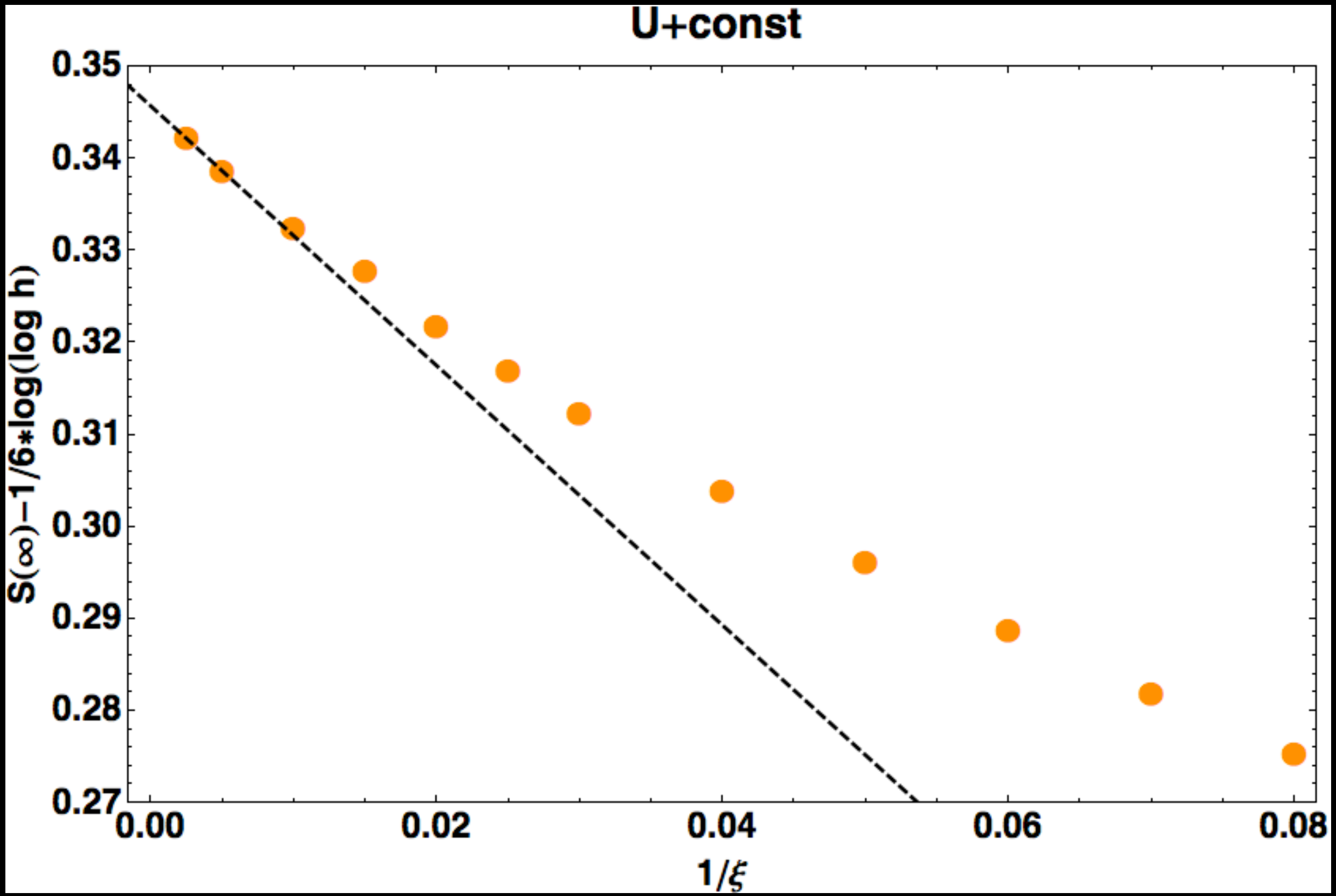
The scaling limit is $H = i\int dx \quad \left(\psi\partial_x\psi - \bar{\psi}\partial_x\bar{\psi} - m\bar{\psi}\psi\right)$

And we can compare results with

$$S_A \sim \frac{c}{3}\log(\log h) + U + const - \frac{1}{\alpha}K_0\left[L\log h\right] + \ldots$$

$c = 0.5$, $U = -0.13198$, $\alpha = 8$

Numerical results:

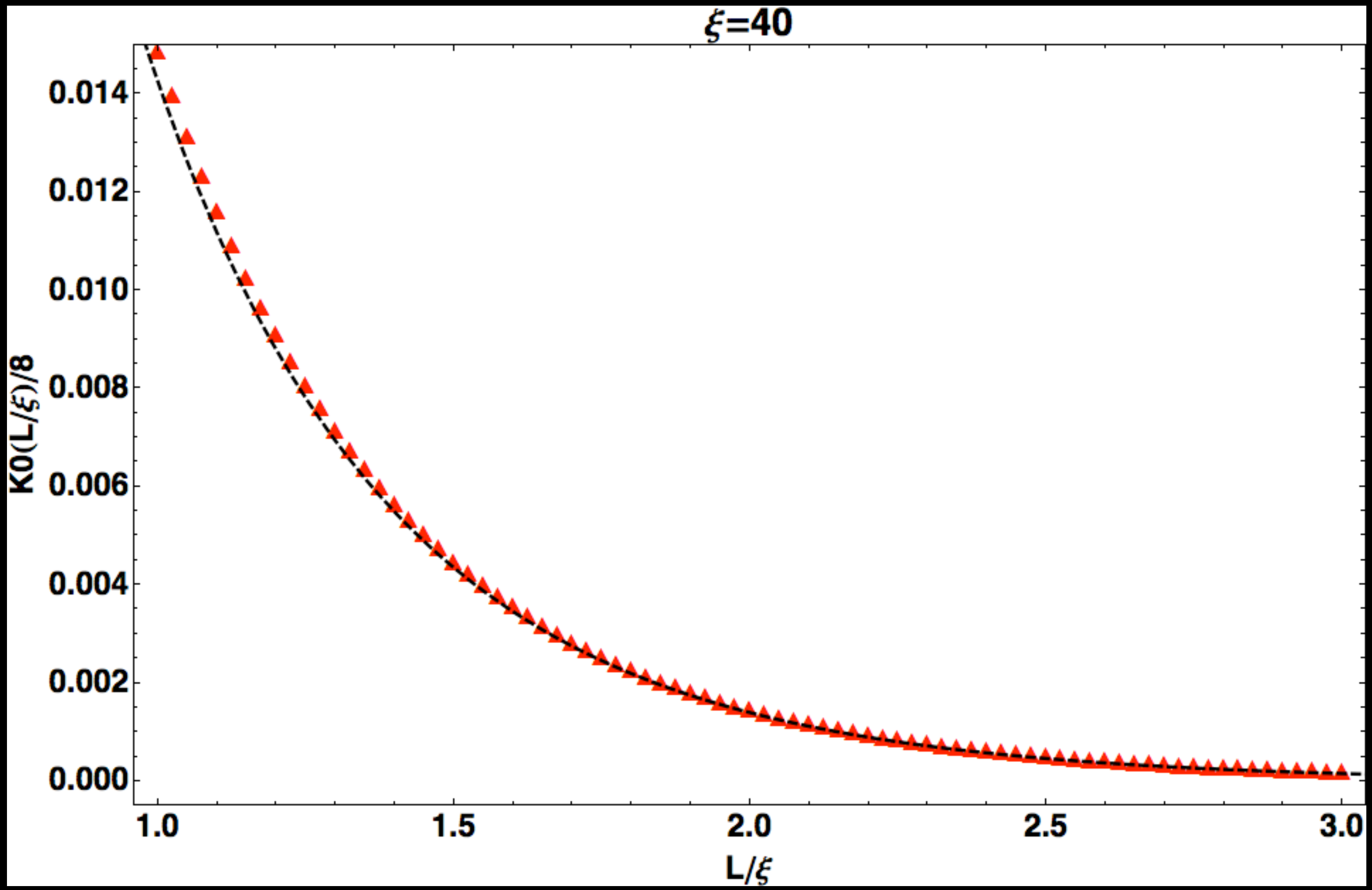


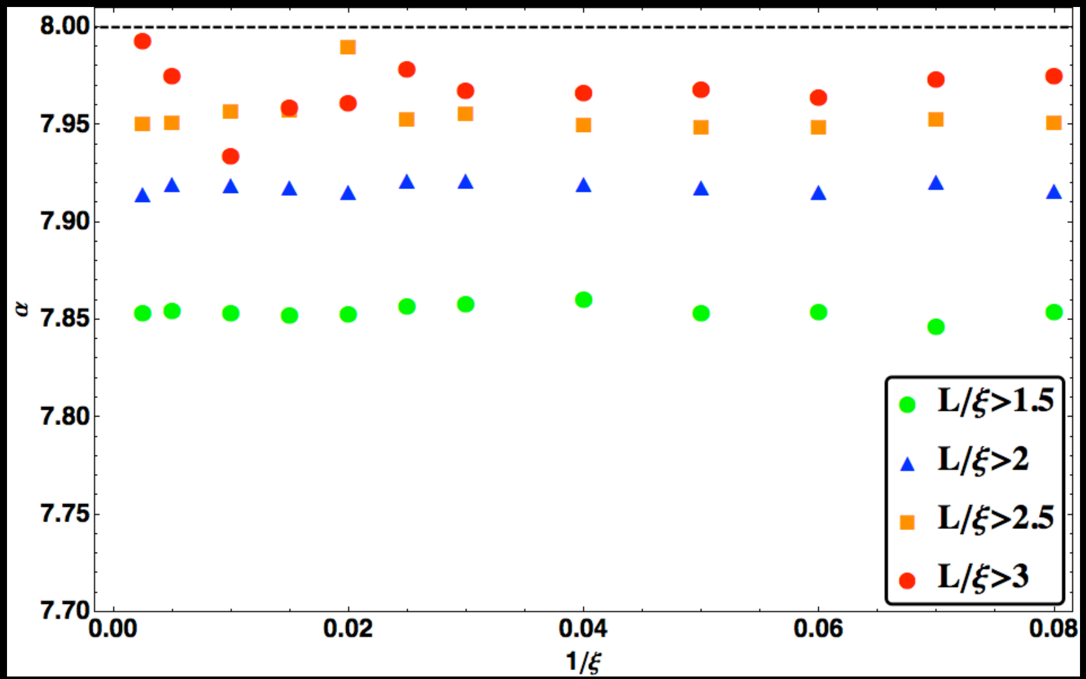
U+const = 0.34657 (Peschel, 2004)

U+const = 0.34565 from fit

U = - 0.13291
0.7% of discrepancy

Numerical results:





Best estimate
$\alpha \cong 7.97$

Numerical results:

Another model we can use to check our results is the XXZ chain

$$H = -J \sum_{i=1}^{N} \left[\frac{1}{2} S_l^+ S_{l+1}^- + \text{H.c.} + \Delta S_l^z S_{l+1}^z \right]$$

Q.C.P. $\Delta = 1$

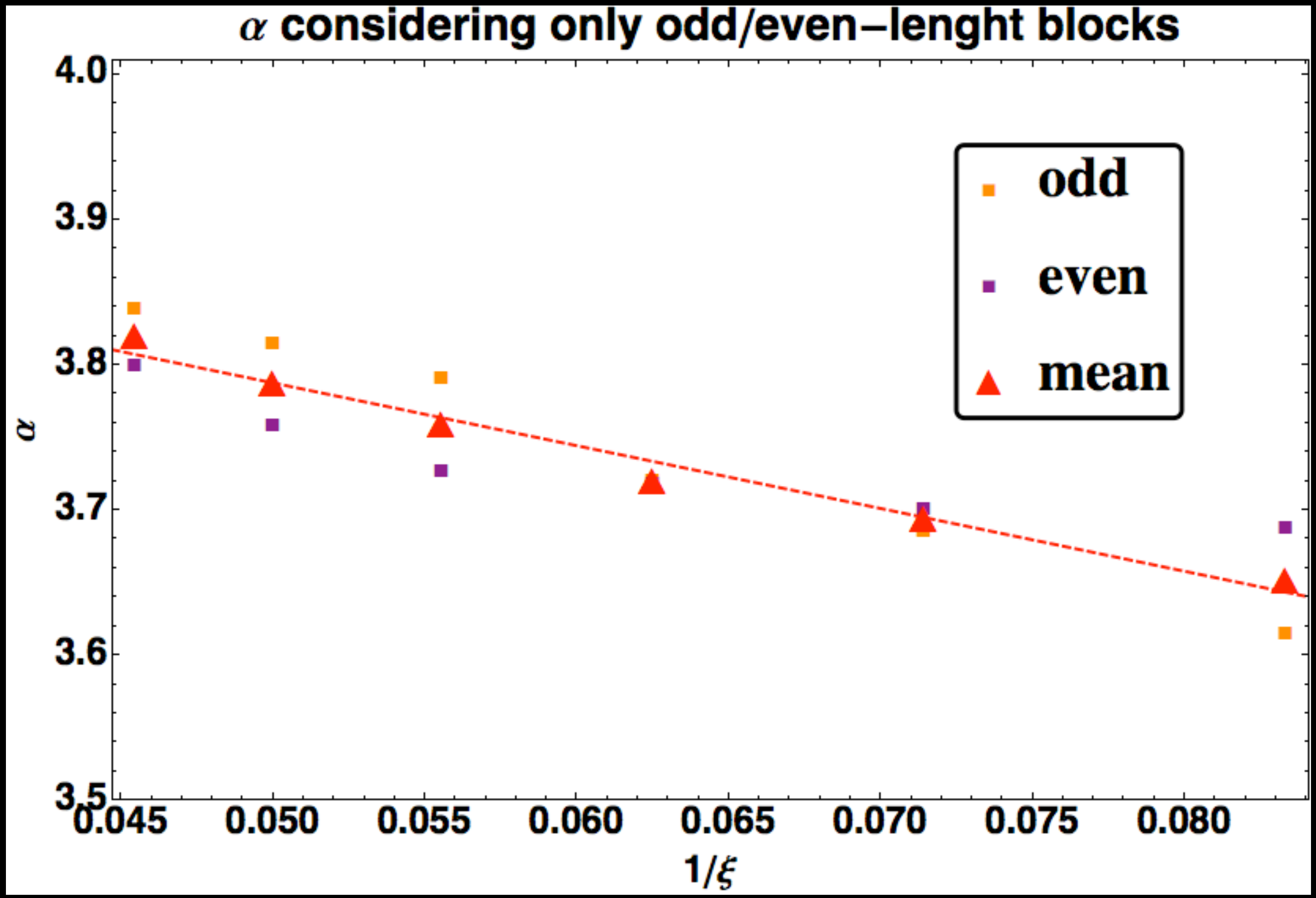
Its scaling theory is sin-Gordon, and we get the correspondence

$$S_A \sim \frac{\overset{1}{\boxed{c}}}{3} \log \xi(\Delta) + \overset{?}{\boxed{U}} + const - \frac{1}{\underset{4}{\boxed{\alpha}}} K_0 \left[L/\xi(\Delta) \right] + \ldots$$

It is an interesting benchmark because of the presence of two lightest particles.

Numerical results:

More involved and we need more sophisticated numerical methods



$\alpha \cong 4.004$

Thank you for listening
I hope you enjoyed it!

A couple of developments:

- Checking the validity of sub-leading term on a non-integrable theory.
- The twist-field method is suitable for evaluations of Renyi entropies, in particular the single-copy entropy...