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Parameter estimation using NOON states over a relativistic quantum channel

This will be appearing on the arXiv soon.

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Contents

Motivation

- Parameter estimation is a process for extracting information about physical quantities.
- NOON states provide improvement over classical limits.
- The universe is both relativistic and quantum.
- Accelerated frames provide an approximation for hovering near black holes.
- We want to understand how we may be able to improve over classical limits, when subjected to relativistic noise.

Cramér-Rao bound

- The Cramér-Rao bound limits the precision of any possible measurement of a continuous parameter.
- This bound states that the expected variance of the parameter has a lower bound set by the Fisher information.

$$\left\langle (\Delta\theta)^2 \right\rangle \geq \frac{1}{M\mathscr{F}(\theta)}$$

- Here, M is the number of measurements made.
- The Fisher information is dependent on both θ and the particular way the state was parametrised.

Braunstein & Caves, PRL. 72. 3439 (1994)

Fisher Information

- We use the quantum Fisher information.

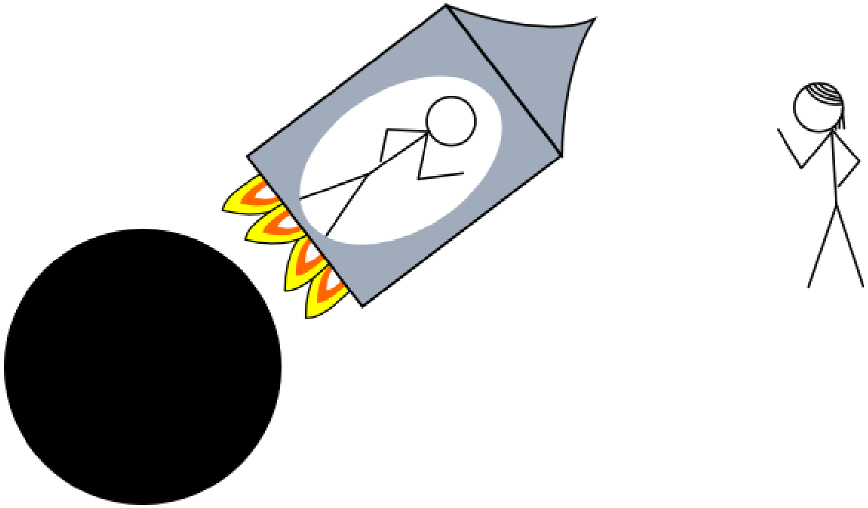
$$\mathscr{F}(\theta) = \mathrm{tr}[\rho' \mathscr{L}_\rho(\rho')]$$

- It is interpreted as the maximum amount of information extractable from a single measurement.
- This calculation involves the lowering superoperator and the derivative with respect to θ.

$$\mathscr{L}_A(B) = \sum_{jk} \frac{2B_{jk}}{(\lambda_A)_j + (\lambda_A)_k} |j\rangle\langle k|$$

- The basis jk is the basis in which A is diagonal.
- The eigenvalues of A written as a vector are λ_A.

Relativistic setup



- Alice is freely falling while Rob maintains his distance to the black hole.
- Rob is approximated with an accelerated observer in flat spacetime.
- Alice encodes and sends the state.
- Rob receives the state and measures to find information about θ.

Transformation

- The parametrised state we use is the standard NOON state in metrology.

$$|\psi\rangle = |N, \varnothing\rangle + e^{iN\theta}|\varnothing, N\rangle$$

- This is encoded as excitations in Unruh modes using an extension of the single rail and dual rail encoding schemes.

$$|\psi\rangle^{(s)} = |\varnothing\rangle + e^{iN\theta}|N\rangle$$

$$|\psi\rangle^{(d)} = |N\rangle_0|\varnothing\rangle_1 + e^{iN\theta}|\varnothing\rangle_0|N\rangle_1$$

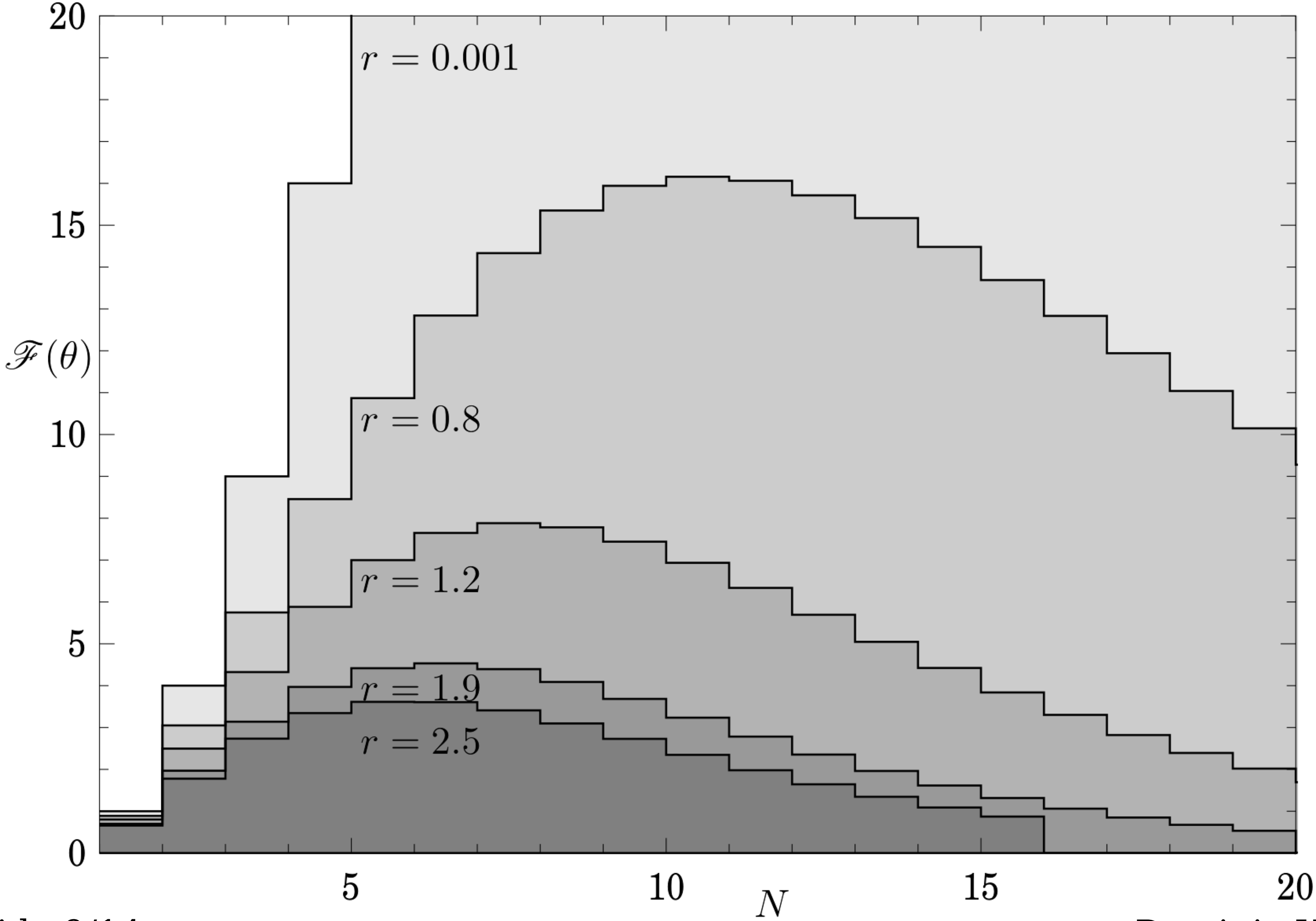
- Unruh modes are unphysical because of delocalization and infinite oscillation near the origin. However, they may provide limiting behaviour because they map directly to single frequency Rindler modes.

Takagi PTPS. 88 (1986)

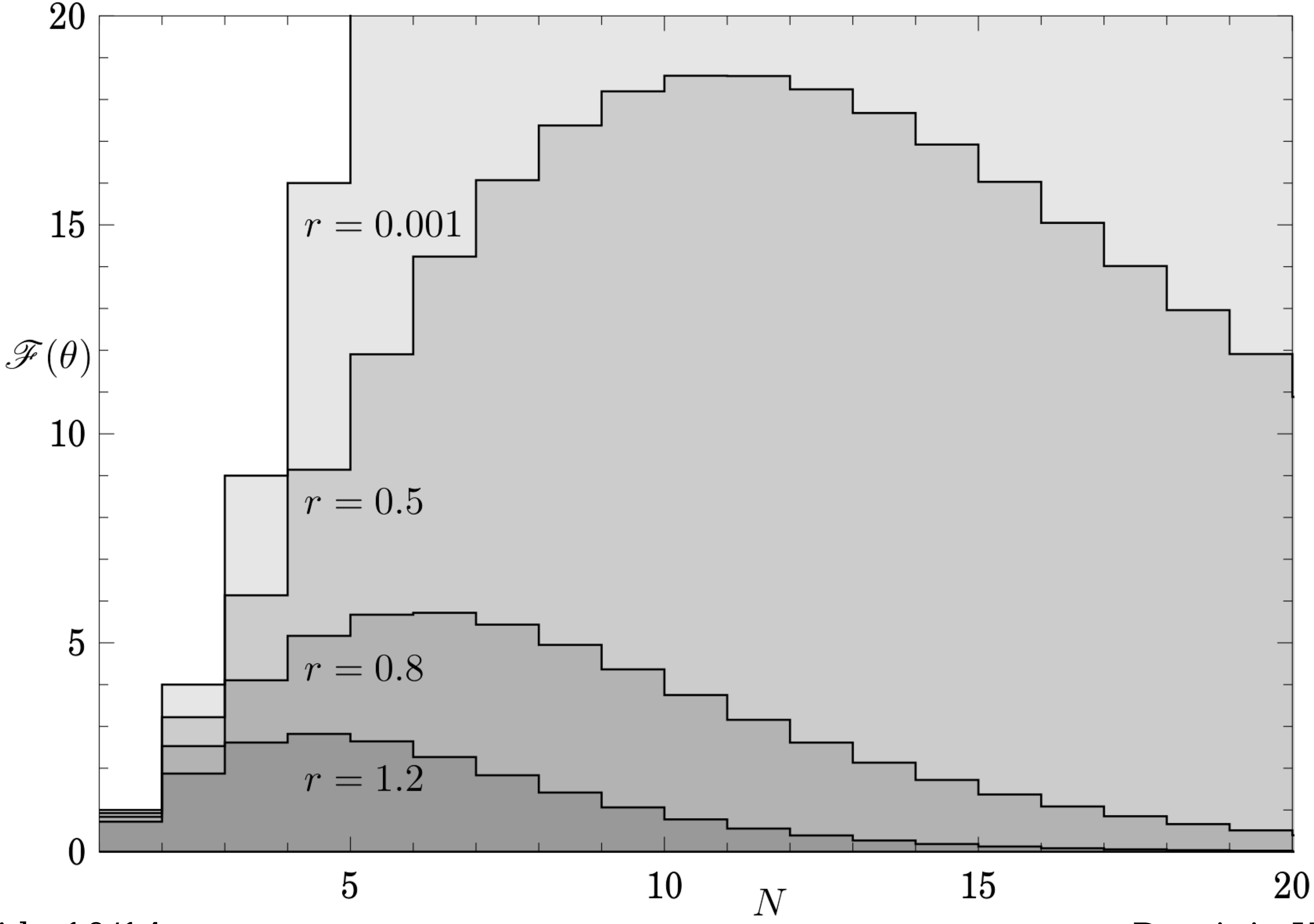
Process

1. Alice creates the appropriate states and encodes them in the field.
2. We transform to both Rob and anti-Rob using two mode squeezing, the appropriate transformation for an accelerated observer.
3. We must trace out the inaccessible modes behind the horizon, in anti-Rob's region of spacetime.
4. This leaves Rob with a partially mixed state containing some thermal noise due to the Unruh-Hawking effect.
5. This state is dependent on, r, the squeezing parameter, N, the number of excitations and θ, the parameter due to be measured.
6. We now calculate the Fisher information; the information that Rob can extract from the state he received about the parameter θ.

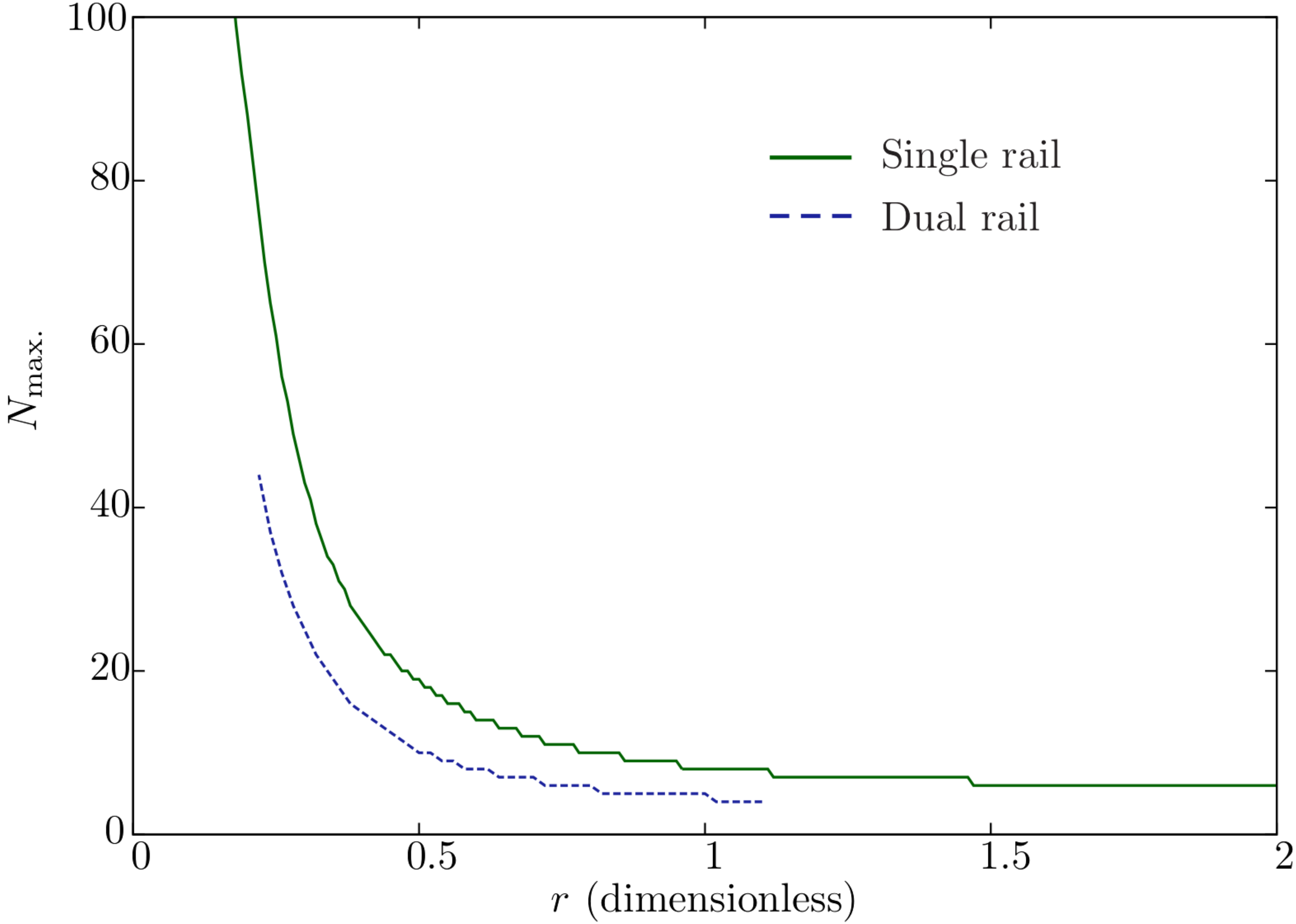
Dependence on N
Single rail



Dependence on N
Dual rail



Maximum N



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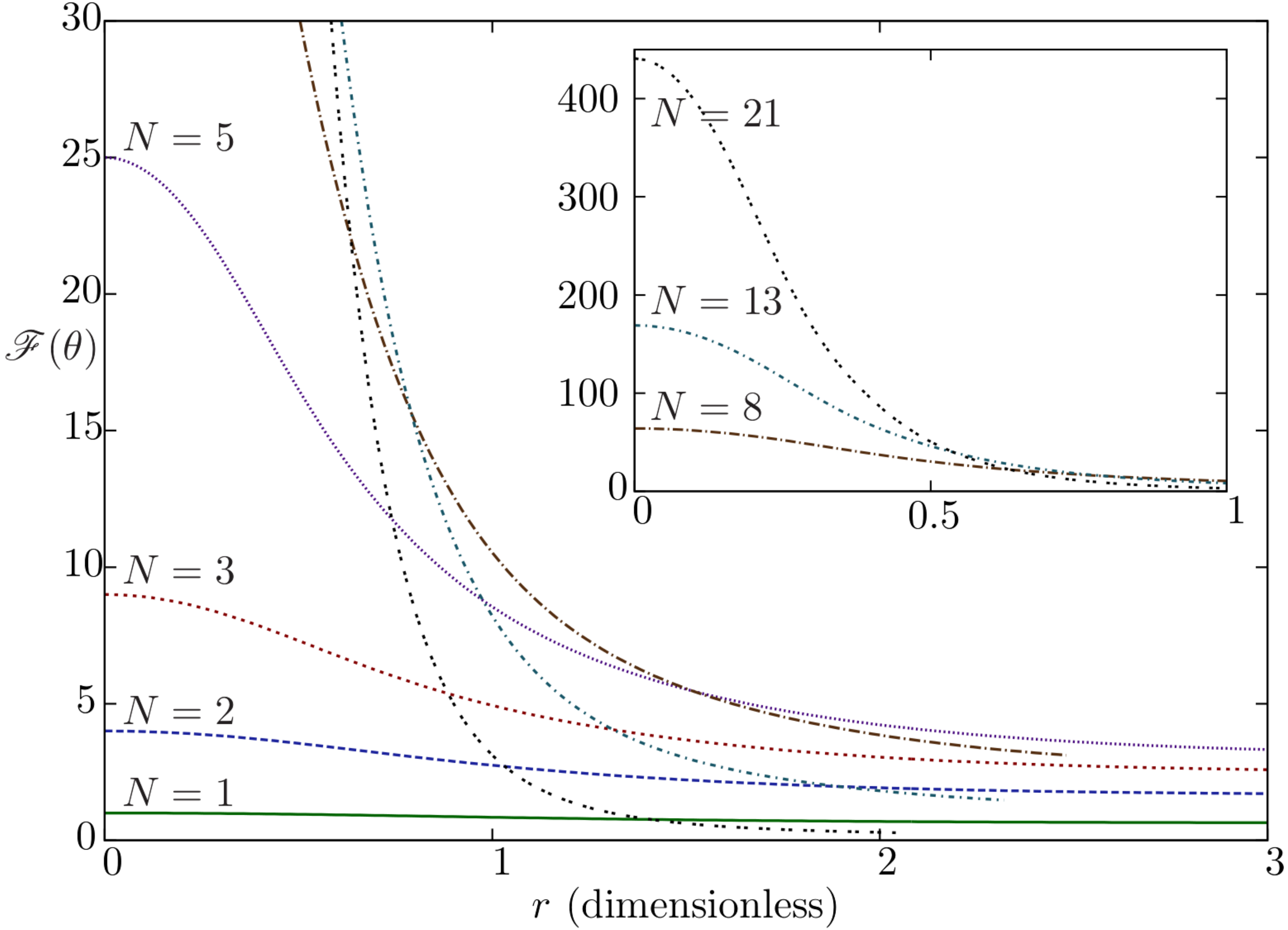
Dependence on r
Single rail



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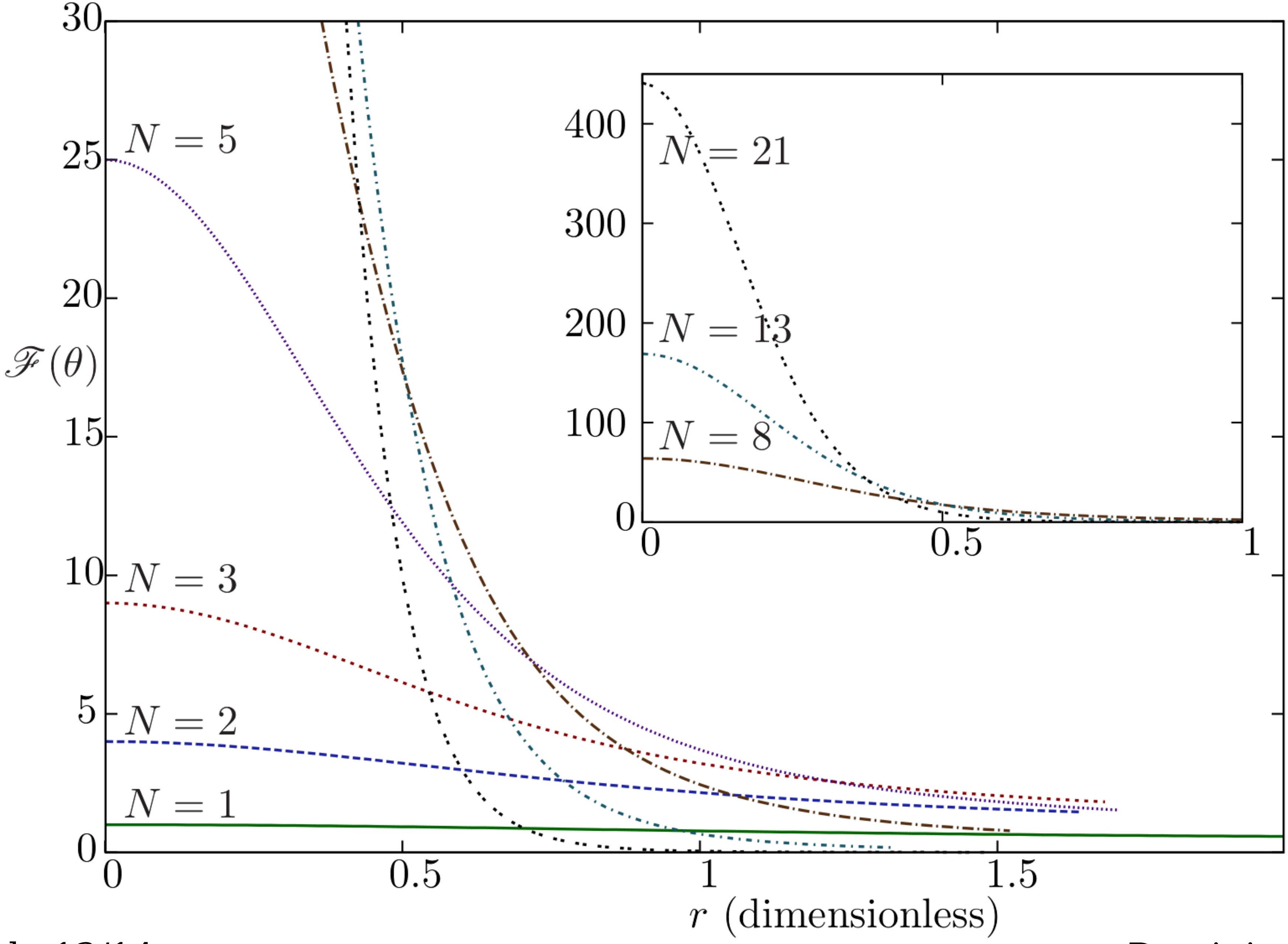
Dependence on r
Dual rail



Conclusion

- At zero noise, the Fisher information rises with N^2.
- We find that the noise is dependent on N, leading to an optimum N for the communication process.
- This also causes the single rail to outperform the dual rail, contrary to previous work.
- The Fisher information drops monotonically with increasing noise.