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A Hitchhiker's Guide to Quantum Field Theory

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Quantum Field Theory (QFT)

- Fusion of quantum mechanics and special relativity.
- Provides a natural framework for particle physics.
- Major successes such as
  - prediction of antiparticles,
  - QED, QCD, standard model
  - amazingly high accuracy with which predictions are made,
  - most recently, discovery of a Higgs-like Boson.
- Axiomatisation scheme: the **Wightman axioms** (1950s).
  $\Longrightarrow$ PCT, Spin and Statistics, and All That (e.g. Haag-Ruelle scattering, Haag's theorem, Reeh-Schlieder theorem).

QFT (continued...)

A **free** sample: The scalar field

- Klein-Gordon equation $(\frac{\partial^2}{c^2\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} + \frac{m^2c^2}{\hbar^2})\phi = 0$.
- "Bosonic" Fock space, physical vacuum = Fock vacuum (unique Poincaré invariant state), and quantised fields obeying the CCRs in terms of annihilation and creation operators.

$\Longrightarrow$ **1** representation of the CCRs!

- Is this the only representation? -No, there are many, many more (even unitarily inequivalent and non-Fock space representations e.g. thermal states)!
- Is there a preferred choice? -Yes, demand a unique Poincaré invariant state of lowest energy, the vacuum! However, this is very much tied to Minkowski space!

Algebraic QFT (AQFT) / Local Quantum Physics (LQP)

[R.Haag & D.Kastler 1964]: QFT in terms of its observables

$\mathcal{O}$ = open subset of Minkowski space with compact closure.

$\mathfrak{A}(\mathcal{O})$ = abstract $(C)^*$-algebra of observables associated with $\mathcal{O}$.

A QFT is characterised by its **net of local observables**

$$\mathcal{O} \longmapsto \mathfrak{A}(\mathcal{O})$$

subject to Haag-Kastler axioms:

- $\mathcal{O}_1 \subseteq \mathcal{O}_2 \Longrightarrow \mathfrak{A}(\mathcal{O}_1) \subseteq \mathfrak{A}(\mathcal{O}_2)$ (isotony).
- $\mathcal{O}_1$ spacelike to $\mathcal{O}_2 \Longrightarrow [\mathfrak{A}(\mathcal{O}_1), \mathfrak{A}(\mathcal{O}_2)] = 0$.
- $\mathfrak{P}_+^\uparrow$ acts by automorphisms, $\alpha_g \mathfrak{A}(\mathcal{O}) = \mathfrak{A}(g\mathcal{O})$.
- $\widehat{\mathcal{O}}$ causal completion of $\mathcal{O}$, $\mathfrak{A}(\widehat{\mathcal{O}}) = \mathfrak{A}(\mathcal{O})$.

States are positive, linear functionals on the **algebra of quasilocal observables** $\mathfrak{A}_{\mathrm{qloc}} = \overline{\bigcup_{\mathcal{O}} \mathfrak{A}(\mathcal{O})}$ $\big((C^*)$-inductive limit$\big)$.

AQFT / LQP (continued...)

Successes and progresses such as

- allows one to deal with all possible, in particular unitarily inequivalent, representations of the algebra of observables at one blow,
  - Pick a(n algebraic) state + GNS-construction = Hilbert space + observables as self-adjoint operators.
- understanding of superselection rules and the role of unobservable fields,
- QFT in curved spacetimes (vandalising Haag-Kastler axioms),
- cosmology,
- perturbative AQFT,
- quantum energy inequalities.

Locally Covariant Quantum Field Theory (lcQFT)

[R.Brunetti, K.Fredenhagen, R.Verch 2003]: Use of category theory

A lcQFT is a **covariant functor** $\mathfrak{A} : \mathbf{Loc} \longrightarrow$ **mono-**$\mathbb{1}$**-(C)*-Alg**.

$$
\begin{tikzcd}
\mathbf{L} \arrow[r, "\varphi"] \arrow[rr, bend left, "\psi \circ \varphi"] \arrow[d, mapsto, "\mathfrak{A}"'] & \mathbf{M} \arrow[r, "\psi"] \arrow[d, mapsto, "\mathfrak{A}"'] & \mathbf{N} \arrow[d, mapsto, "\mathfrak{A}"] \\
\mathfrak{A}(\mathbf{L}) \arrow[r, "\mathfrak{A}(\varphi)"] \arrow[rr, bend right, "\mathfrak{A}(\psi \circ \varphi)"'] & \mathfrak{A}(\mathbf{M}) \arrow[r, "\mathfrak{A}(\psi)"] & \mathfrak{A}(\mathbf{N})
\end{tikzcd}
$$

$\mathfrak{A}(\mathrm{id}_{\mathbf{M}}) = \mathrm{id}_{\mathfrak{A}(\mathbf{M})}$, $\mathfrak{A}(\psi \circ \varphi) = \mathfrak{A}(\psi) \circ \mathfrak{A}(\varphi)$

lcQFT (continued...)

a **locally covariant quantum field** is a **natural transformation**

$$\begin{tikzcd}
\mathscr{D}(\mathbf{M}) \arrow[r, "\Phi_{\mathbf{M}}"] \arrow[d, "\mathscr{D}(\psi)"'] & \mathfrak{A}(\mathbf{M}) \arrow[d, "\mathfrak{A}(\psi)"] \\
\mathscr{D}(\mathbf{N}) \arrow[r, "\Phi_{\mathbf{N}}"] & \mathfrak{A}(\mathbf{N})
\end{tikzcd}$$

in the category of topological spaces.

- A change of the target category allows to describe other physical theories in a locally covariant fashion.

lcQFT (further continued...)

The functorial framework is **not** a mere reformulation of known results but a **powerful ally** as it stresses the physical features of a theory in a background independent way. It has led to major progresses in many areas, e.g.

- recovery of AQFT,
- spin-statistics in curved spacetimes,
- cosmology,
- quantum energy inequalities,
- Reeh-Schlieder theorem in curved spacetimes,
- perturbative AQFT,
- approaches to Quantum Gravity,
- classical field theory,
- what it means to represent the same physics in all spacetimes (SPASs).

Development of perturbative AQFT in little more detail

- standard QFT: Rules to obtain the quantities of interest of an interacting quantum field in terms of perturbation series. To deal with infinities, **renormalisation** is employed $\Longrightarrow$ **renormalisation parameters**.
- In curved spacetimes, we need **Wick polynomials** and their **time ordered products**.
- AQFT: Wick polynomials and time ordered products in curved spacetimes using L.Hörmander's microlocal analysis and Epstein-Glaser renormalisation but: renormalisation **functions** (with quite arbitrary dependence on the spacetime points).
- lcQFT: Reduction of the renormalisation functions to finitely many renormalisation parameters, whose number is fixed by the same sort of power counting as in Minkowski space.

So long, and thanks for all the fish!
(and, of course, for your attention)

(http://en.wikipedia.org/wiki/File:Inia.jpg)