Detector for a massless (1+1) field: Hawking effect without infrared sickness

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Outline

Motivation

What do we want to do?

We'd like to examine thermality in time-dependent situations. In particular, what does one feel as one falls into a BH? Under which circumstance can we talk about thermality?

Why $1+1$ spacetimes?

- Great simplification (e.g. no backscattering in Schwarzschild geometry).
- Qualitatively similar.

What is a particle detector?

(3+1) Unruh-DeWitt detector

Two-level system coupled to a scalar field. Heuristically, can think of an atom interacting with a scalar field by absorbing or emitting field quanta.

The Hamiltonian is $H = H_D + H_\phi + H_{\text{int}}$, where

$$H_{\text{int}} = c\mu(x(\tau))\phi(x(\tau)),$$

where c is some coupling constant, $\mu(\tau)$ is the monopole moment of the detector, $\phi(\tau)$ is a scalar field, and τ the detector's proper time. For uniformly accelerated trajectory $\Rightarrow$ Unruh effect, i.e.,

$$\dot{F}(\omega) = \omega \frac{1}{e^{2\pi\omega/a} - 1} \leftarrow \text{Thermal spectrum}$$

Unruh effect

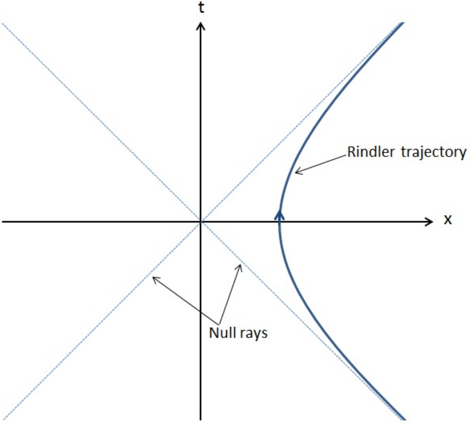

Fig 1. Detector following a Rindler trajectory feels bath of particles at temperature $T = \frac{\hbar a}{2\pi c k}$.

The Unruh-DeWitt detector in 1+1

- Transition probability = (Detector internal properties) $\times$ (Response function), i.e. $P(\omega) \propto F(\omega)$.
- Transition rate, $F/\Delta\tau$, goes like Fourier transform of the so-called Wightman function:

$$F/\Delta\tau = \int_{\infty}^{\infty} d\tau' \, \mathrm{e}^{-\mathrm{i}\omega(\tau-\tau')} W(\tau,\tau'),$$

where $W(\tau,\tau') \equiv \langle 0|\phi(\tau)\phi(\tau')|0\rangle$.

- But Hadamard $W(\tau,\tau')$ only exists for massive fields in 1+1 dimensions!

The derivative coupling detector

- Infrared divergence for massless fields $\sim \log(m)$!
- Is the Unruh-DeWitt detector natural in 1+1 dimensions?
- Introduce a derivative coupling detector.
- Suggested by Don Marolf and first mentioned in the literature at least in (Davies and Ottewill, 2002).

The derivative coupling detector

Consider

$$H_{\text{int}} = c\chi(\tau)\mu(\tau)\dot{x}^a\partial_a\phi(x(\tau)).$$

where $\chi(\tau)$ is a smooth switching function of compact support controlling the interaction time. Then

$$F(\omega) = \int_{\infty}^{\infty} d\tau \int_{\infty}^{\infty} d\tau' \, \chi(\tau)\chi(\tau')\mathrm{e}^{\mathrm{i}\omega(\tau-\tau')}A(\tau,\tau'),$$

where $A(\tau,\tau') \equiv \frac{d}{d\tau}\frac{d}{d\tau'}W(\tau,\tau')$ is finite for a massless 1+1 scalar field.

Smooth switching and sharp switching limit

$$\chi(u) = h_1\left(\frac{u - \tau_0 + \delta}{\delta}\right) \times h_2\left(\frac{-u + \tau + \delta}{\delta}\right),$$

with $h_1(x)$ and $h_2(x)$ smooth non-negative functions that satisfy $h_1 = h_2 = 0$ for $x < 0$ and $h_1 = h_2 = 1$ for $x > 1$.

Fig 2. Interaction gets switched on (off) during a vey small time interval δ at time $\tau_0 - \delta$ (τ).

- Switching function allows for time-dependent measurements.
- It also allows to rid of regulator ($\mathrm{i}\epsilon$) (Satz, 2007).

Smooth switching and sharp switching limit

Removing $\mathrm{i}\epsilon$

Hadamard dictates (Décanini and Folacci, 2008):

$$W(\tau', \tau'') = -\frac{1}{4}\left(\sum_{i=0}^{\infty} v_i s^i\right) \log(-s^2),$$

where $s = \tau - \tau'$, and with $v_0 = -\Delta^{1/2}(x(\tau), x(\tau'))$, and Δ is the Van Vleck-Morette determinant.

Smooth switching and sharp switching limit

Removing $\mathrm{i}\epsilon$

$$F(\omega) = 2\int_{-\infty}^{\infty} d\tau \int_0^{\infty} ds \, \mathrm{Re}\, Q'(\tau)\bar{Q}'(\tau - s)[W(\tau, \tau - s) + \mathrm{i}\frac{1}{4} + \frac{1}{2\pi}\log(s)] + F_{\text{const.}} + F_{\log},$$

where $Q(\tau) = \chi(\tau)\exp(-\mathrm{i}\omega\tau)$.

- Then integrate $F_{\text{const.}}$ and $F_{\log}$ once and for all.
- $\mathrm{i}\epsilon$ plays no role in these integrals. Regularisation in χ.

Smooth switching and sharp switching limit

Formally, as $\delta/(\tau - \tau_0) \to 0$, a general formula, *independent* of the details of the switching, may be obtained for the transition rate:

$$\dot{F}(\omega) \equiv dF/d\tau = 2\int_0^\infty ds\, \mathrm{Re}\left[\mathrm{e}^{-\mathrm{i}\omega s} A(\tau, \tau - s) + \frac{1}{2\pi s^2}\right] - \frac{1}{2}\omega.$$

Derivative coupling detector tests

- Thermality: Detector feels the Unruh effect along the Rindler trajectory.
- Limit $m \to 0$ commutes with integral in transition rate formula for massive detectors $\Rightarrow$ can talk about **massless** fields!

Star collapse model: receding mirror

A spacetime in which the boundary moves following the trajectory $v(u) = -\frac{1}{\kappa} \log(1 + e^{-\kappa u})$ (lightcone coods.) emulates the spacetime of a collapsing star. Consider an inertial detector:

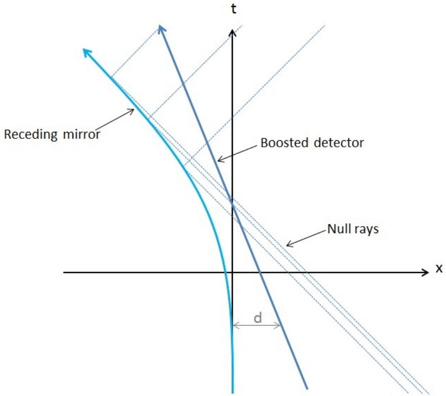

Fig 3. Inertial detector in a Receding mirror spacetime.

Receding mirror: The Wightman function

For the field quantisation, the dynamical boundary dictate a field expansion as

$$\phi(u,v)_{MHS} = \int dk \left(a\, u_{k\,MHS} + a^\dagger\, u^\dagger_{k\,MHS} \right),$$

where

$$u_k = \mathrm{i}(4\pi\omega)^{-1/2}\left[\mathrm{e}^{-\mathrm{i}\omega v} - \mathrm{e}^{-\mathrm{i}\omega p(u)}\right],$$

and $p(u)$ is obtained by demanding that the field vanish at the mirror's trajectory. This leads to $p(u) = -\log(1+\mathrm{e}^{-\kappa u})/\kappa$. So,

$$\begin{aligned} W_\epsilon(x,x') &= \langle 0|\phi(x)\phi(x')|0\rangle \\ &= -(1/4\pi)\log[(p(u)-p(u')-\mathrm{i}\epsilon)(v-v'-\mathrm{i}\epsilon)] \\ &+ (1/4\pi)\log[(v-p(u')-\mathrm{i}\epsilon)(p(u)-v'-\mathrm{i}\epsilon)]. \end{aligned}$$

Receding mirror

- Cannot be solved analytically in full generality. Can consider asymptotic limits.
- Early time measurements ($\tau \to -\infty$).
- Late time measurements ($\tau \to +\infty$).

Receding mirror: Asymptotics

A boosted detector follows the trajectory $x^a = \gamma(\tau, -\beta\tau)$, where $\gamma = 1/\sqrt{1-\beta^2}$.
In the asymptotic past the transition rate is dominantly

$$\dot{F}(\omega)_{\tau \to -\infty} = -\omega \left[1 - \frac{1+\beta}{1-\beta} \cos\left(\frac{2\beta\tau}{1-\beta}\omega \right) \right] \Theta(-\omega). \qquad (1)$$

In the asymptotic future the leading contribution is

$$\dot{F}(\omega)_{\tau \to +\infty} = -\frac{\omega}{2}\Theta(-\omega) + \frac{\omega}{2} \frac{1}{\exp\left[\frac{2\pi\omega}{\kappa} \frac{\sqrt{1-\beta}}{\sqrt{1+\beta}} \right] - 1}. \qquad (2)$$

The 1+1 Schwarzschild BH

1+1 Metric:

$$ds^2 = -(1 - 2M/r)du dv = -[2M \exp(-r/2M)/r] d\bar{u} d\bar{v}$$

Field quantisation is not unique, then vacuum state is not unique!
Three natural vacua:

Vacuum	Wightman function
$\vert 0_B\rangle$ (Boulware)	$W_B(x, x') = -\frac{1}{4\pi}\log[(\Delta u - \mathrm{i}\epsilon)(\Delta v - \mathrm{i}\epsilon)]$
$\vert 0_H\rangle$ (Hartle-Hawking)	$W_H(x, x') = -\frac{1}{4\pi}\log[(\Delta \bar{u} - \mathrm{i}\epsilon)(\Delta \bar{v} - \mathrm{i}\epsilon)]$
$\vert 0_U\rangle$ (Unruh)	$W_U(x, x') = -\frac{1}{4\pi}\log[(\Delta \bar{u} - \mathrm{i}\epsilon)(\Delta v - \mathrm{i}\epsilon)]$

The 1+1 Schwarzschild BH: Static detector

Consider a detector at fixed $r = R$.

Vacuum	Transition rate, $\dot{F}(\omega)$
$\mid 0_B\rangle$ (Boulware)	$-\omega\Theta(-\omega)$
$\mid 0_H\rangle$ (Hartle-Hawking)	$\omega\frac{1}{\exp[8\pi M\omega\sqrt{1-2M/R}]-1}$
$\mid 0_U\rangle$ (Unruh)	$-\frac{\omega}{2}\Theta(-\omega)+\frac{\omega}{2}\frac{1}{\exp[8\pi M\omega\sqrt{1-2M/R}]-1}$

Thermality appears with $T = \kappa(1-2M/R)^{-1/2}/2\pi k_B$, where $\kappa = 1/4M$ is the surface gravity of the Black hole.

The 1+1 Schwarzschild BH: Asymptotics

Consider an infalling detector following a timelike geodesics

- Cannot be solved analytically in full generality. Can consider asymptotic limits.
- Early time measurements ($\tau \to -\infty$).
- Can be done for freely falling detector or boosted detector in the past timelike infinity.

The 1+1 Schwarzschild BH: Conformal diagram

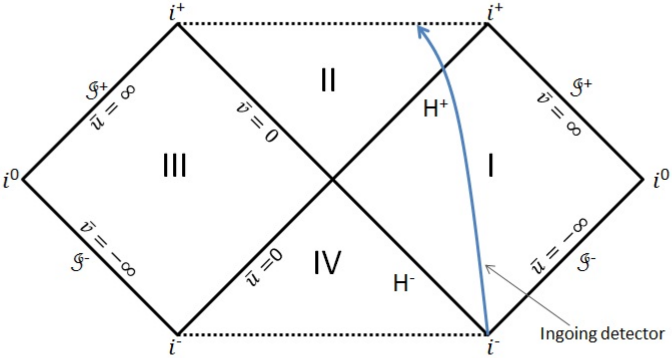

Fig 4. Infalling detector along a geodesic.

The 1+1 Schwarzschild BH: Asymptotics

Consider a detector falling with initial velocity β at $\tau \to -\infty$. A measurement in the asymptotic past yields:

Vacuum	Transition rate, $\dot{F}(\omega)_{\tau \to -\infty}$
$\mid 0_B\rangle$ (Boulware)	$-\omega\Theta(-\omega)$
$\mid 0_H\rangle$ (Hartle-Hawking)	$\frac{\omega/2}{\exp\left[2\pi\omega/\sqrt{\frac{1+\beta}{1-\beta}}\frac{1}{4M}\right]-1} + \frac{\omega/2}{\exp\left[2\pi\omega/\sqrt{\frac{1-\beta}{1+\beta}}\frac{1}{4M}\right]-1}$
$\mid 0_U\rangle$ (Unruh)	$-\frac{\omega}{2}\Theta(-\omega) + \frac{\omega}{2}\frac{1}{\exp\left[2\pi\omega/\sqrt{\frac{1+\beta}{1-\beta}}\frac{1}{4M}\right]-1}$

Thermality appears with a blue(red)shifted factor in the temperature, $T = \sqrt{\frac{1\pm\beta}{1\mp\beta}}\frac{1}{4M}/2\pi k_B$.

Numerics: Hartle-Hawking

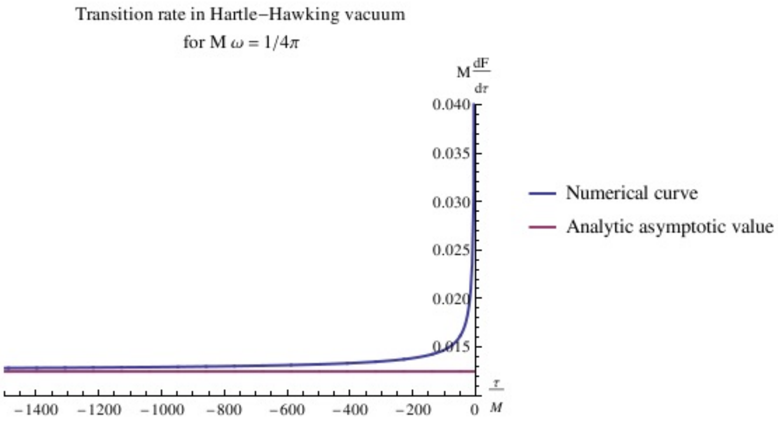

Fig 5. Asymptotic behaviour of in the Hartle-Hawking vacuum. Schwarzschild radius, $r/M = 2$, at $\tau/M = -4/3$. Singularity at $\tau = 0$.

Numerics: Unruh ω dependence

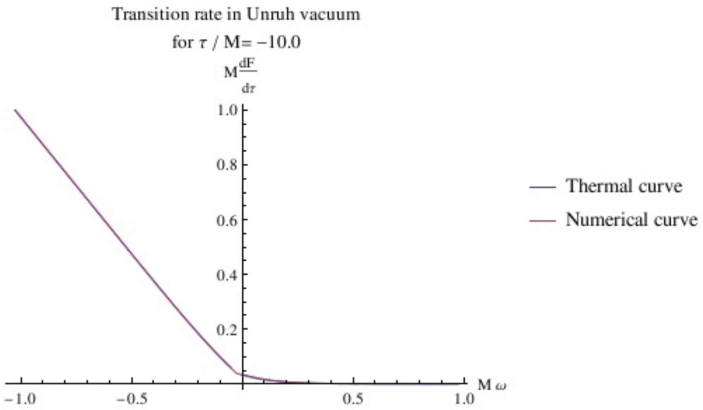

Fig 6. ω-dependence in the Unruh vacuum. $\tau/M = -10$, $r/M = 2(15/2)^{2/3} \approx 7.66$.

Numerics: Unruh ω dependence

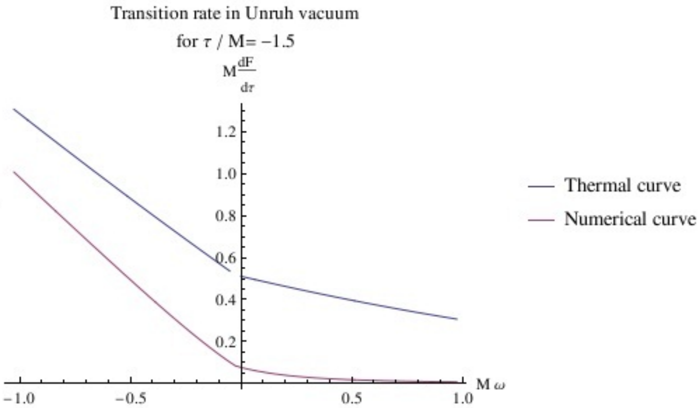

Fig 7. ω-dependence in the Unruh vacuum.
$\tau/M = -3/2$, $r/M = 2(9/8)^{2/3} \approx 2.16$.

Current work

What are we doing now?

- One can no longer talk about a thermal spectrum for an infalling detector in the inside region! (Tolman factor and see plots.)
- **Disagreement** with a **thermal** spectrum (weighted with appropriate blue(red) shifts and Tolman factors) along an infalling geodesic **increases** in the proximity of the horizon and strongly depends on ω.

Thanks for your attention!

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